STUDY ON THE CAPILLARY ADHESIVE FORCES BETWEEN SOLID PARTICLES WITH A LIQUID LAYER AT THE POINTS OF CONTACT

1. SPHERICAL PARTICLES

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Three-phase systems consisting of solid particles, pores, and a liquid phase are encountered in practice in various fields of engineering and technology. Examples are flotation systems, moist soils, some metallurgical materials. As shown previously [1, 6], the capillary properties of these systems may influence the sintering and compacting of cermets in an essential way.

The liquid by which solid particles are held together exerts a squeezing force which acts during the entire period of sintering and causes the systems to contract. Calculations show that, depending on the size of the solid particles and the surface tension of the liquid metal, the adhesive pressure may attain values of ten, twenty and even hundreds of decanewtons per square centimeter.

In the present study we investigated the simplest capillary model system consisting of spherical particles with gas and liquid phases between them. Two spherical particles with a ring of liquid around the point of contact are the units from which such a system is built up.

If the liquid wets the particle surface sufficiently, it gives rise to a capillary attraction which squeezes the particles and causes them to make contact. This attraction depends on the particle shape, the properties of the liquid surface, and many other factors. We determined the dependence of the compressive force on the amount of liquid. This problem was solved theoretically and experimentally.

Calculation of the Compressive Forces. The capillary attraction by which the particles are held together consists of the following two components:

1. The pressure drop over the surface produced by the curvature of the liquid meniscus. This drop is determined by Laplace's first law, which in our case reads

\[ \Delta P = \sigma_1 \left( \frac{1}{\rho_2} - \frac{1}{\rho_1} \right), \]  

where \( \sigma_1 \) denotes the surface tension of the liquid, and \( \rho_1 \) and \( \rho_2 \) are the radii of curvature of the liquid ring (Fig. 1). Since the pressure inside the liquid will be lower, the particles are exposed to the compressive force

\[ F_1 = \Delta P \cdot S, \]

where \( S \) is the projection of the liquid-solid boundary area onto a plane perpendicular to the direction of the force.

2. Each particle of the pair examined can be considered to be partly immersed in the liquid. In this case a force determined by the projection of the surface tension vector onto the \( \mathbf{OO}_1 \) direction (Fig. 1) will act on the perimeter where the liquid contacts the particle; the magnitude of this second component of the compressive force acting on the two particles will be:

\[ F_2 = l \cdot \sigma_1 \sin (\varphi + \theta), \]

where \( l \) denotes the length of the perimeter of wetting. We note that the latter term, which, as calculations have shown, is of considerable magnitude if the amounts of liquid are large, was ignored in paper [2].
Total force by which the particles are squeezed together equals:

\[ F = F_1 + F_2 = \sigma \left[ \pi R^2 \sin^2 \varphi \left( \frac{1}{\theta_2} - \frac{1}{\theta_1} \right) + 2\pi R \cdot \sin \varphi \cdot \sin (\varphi + \theta) \right], \tag{3} \]

where

\[ \theta_1 = R \cdot \sin \varphi - R \left( 1 - \cos \varphi \right) \frac{1 - \sin (\varphi + \theta)}{\cos (\varphi + \theta)} \]

and

\[ \theta_2 = R \cdot \frac{1 - \cos \varphi}{\cos (\varphi + \theta)} ; \]

these expressions are based on the assumption that arc AA_1 is a circular arc with radius \( \rho \). In the case of complete wetting, we must substitute \( \theta = 0 \) in these formulas. In the latter case, the volume of the liquid ring is given by

\[ V = 2\pi R^3 \left( \sec \varphi - 1 \right)^2 \left[ 1 - \left( \frac{\pi}{2} - \varphi \right) \tan \varphi \right]. \tag{4} \]

The gravitational field, which will change the curvature of the surface of entrapped liquid, was ignored in the derivation of formula (4). Hence, it will hold only either at small angles \( \varphi \) (small amount of liquid), or at any \( \varphi \), provided that the particles are so small that the distorting effect of gravity can be ignored.

To measure the adhesive forces between two spherical particles, we designed and constructed the special equipment shown in Fig. 2a, b. The spheres 1 were suspended by the wires 2, which are fixed to the slider nuts 3 with left- and right-hand threads; the slider nuts are displaced by turning the microscrew 4 having left- and right-hand threads. The distance between the slider nuts (the distance between the points where the wires are attached) is measured by means of a scale.

The adhesive force between the two spheres is found from the formula:

\[ F = P \tan \alpha = P \cdot \frac{S - 2R}{2 \sqrt{\rho - \left( \frac{S - 2R}{2} \right)^2}}, \]

where \( P \) is the weight of the sphere, \( l \) the length of the suspension wire (the distance between the point where it is attached to the slider nut and the center of the sphere), \( S \) the maximum distance between the slider nuts, \( R \) the radius of the sphere. During a measurement the distance between the slider nuts is increased until the spheres are drawn