The authors examined special features of the stress–strain state in pressed and welded panels enforced with stiffness. Relations for characterizing the cracking resistance in axial and biaxial loading are derived. The calculated data are in agreement with the experimental values.

Practical experience with service of panels reinforced with stiffness shows that main cracks propagating in them decelerate on approaching stiffness and then branch out (Fig. 1).

We shall examine propagation of a crack in a panel taking into account the uniaxial and biaxial mode of loading and also of heterogeneities of the stress fields both away from the stiffener and on approach to it. Since the thickness of the panels is comparatively small (5-10 mm), to solve conventional problems of sheet theory we shall assume that they contain a plane stress state, i.e.,

\[
\begin{align*}
\sigma_{xz} &= \sigma_{yz} = \sigma_{zz} = 0; \\
\sigma_{xx} &< 0; \quad \sigma_{yy} > 0; \quad \sigma_{xy} > 0.
\end{align*}
\]

The stress–strain state (SSS) is differentiated more accurately in fracture mechanics, taking into account the nonuniformity of the field, including in respect of thickness. This results in distortion of the crack front. In the tensile loading a sheet of small thickness the cross section ahead of the crack greatly decreases.

Joining a stiffener prevents reduction of the thickness of the sheet. In this case an additional component of the stress state \(\sigma_{zz}\) forms and causes constraint of longitudinal strains \(\varepsilon_{xx}\) so that the stress state changes to a different, more complicated form. When a crack approaches the stiffener (if the supplied energy is unchanged and the new component \(\sigma_{zz}\) appears), the vector of the energy sink at the tip of the crack changes its form. This is also indicated by the variation of the form of the plasticity zone at the crack tip (Fig. 2). Thus, the supplied energy is used for widening the crack front and transforming it from continuous to surface (Fig. 3). This results in crack deceleration of the direction in its main propagation direction.

We shall examine the energy aspect of widening of the crack front (and of its propagation). The energy hypothesis of strength links the start of a critical stress state in the material of the structure with build up of the internal energy up to some threshold value characterized by stress intensity.

The energy of the unit volume of the panel is

\[
W = \frac{1}{2E} \left| \sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - 2\mu \left( \sigma_{xx} \sigma_{yy} + \sigma_{xx} \sigma_{zz} + \sigma_{yy} \sigma_{zz} \right) \right|, \tag{2}
\]

1. Uniaxial Loading, Pressed Panel. It is well known that the surface energy of a crack is equal to the product of the surface area of the crack by the energy used per unit surface of the crack. The critical load will be evaluated by reducing it to the rate of energy release along crack length. This is equivalent to the operation in which the sheet is examined on the basis of unit thickness in which the crack area is identical with its length (\(h_{\text{conv}} = 1\)).
Fig. 1. Panel with a crack.

Fig. 2. Form of the plasticity zone at the crack tip.

Fig. 3. Form of the crack front away from the stiffener (1) and close to it (2).

\[ f = 2hl ; \quad f = h \int_{0}^{2l} \delta(y) \, dy , \]  
\[ (3) \]

where \( f \) is the crack area; \( l \) is the crack half-length; \( h \) is the sheet thickness; \( \delta(y) \) is the ratio of the transverse dimension of the crack to sheet thickness.

We shall write the general condition for crack propagation:

\[ \frac{\partial (\Delta \nu)}{\partial f} = 4\gamma . \]  
\[ (4) \]

Here \( \gamma \) is the surface energy of the crack; \( \Delta \nu \) is the variation of the potential as a result of the presence of the crack, \( \Delta \nu = \lambda_0 \sigma_0^2 \sigma_0^2 / \gamma \), where \( \lambda_0 \) is a multiplier depending on Poisson's coefficient, \( \sigma_0 \) is the stress applied at infinity. Consequently, we had

\[ f = \frac{2E\gamma}{\lambda_0 \sigma_0^2} ; \]  
\[ (5) \]

and

\[ \sigma_0^2 h \int_{0}^{2l} \delta(y) \, dy = \text{const} \, c , \quad c = \frac{2E\gamma}{\gamma_0} . \]  
\[ (6) \]

The dependence (6) is shown in Fig. 4 (curve 1). For the case with \( \delta(y) = 1 \) (curve 2) Eq. (6) gives the conventional dependence to \( 2l = f(\sigma_0) \).

We shall examine on the basis of this approach the main integral relationship of the theory of cracks by adding, in accordance with Eq. (3), a new meaning to the crack characteristic \( l \) and replacing it by \( f \). In accordance with [1] in this case we can write