Many applications in technology require structural materials capable of working for long periods of time at temperatures above 1300°C. The refractory compounds which can be used to produce these materials are brittle. It is therefore essential to develop methods of increasing crack resistance. Several methods are available for inhibiting crack propagation in brittle materials [1, 2]. These methods include the introduction into the structure of the material of a second, tougher phase in the form of disperse particles or reinforcing fibers, the formation of a microcracking structure or fields of internal stresses which complicate the trajectory of crack propagation, etc. The aim of these methods is to ensure stable crack propagation during the fracture of the material.

Stable crack growth in the brittle single-phase materials can take place both due to thermally activated processes and as a result of the instantaneous reaction of the system to the external disturbing action in active loading. The concept of the critical stress intensity factor $K_{IC}$ cannot explain stable crack growth in these two cases.

The role of the thermally activated processes in brittle fracture has been examined in a number of investigations (see, e.g., [3]). In the present article, the authors describe the results of investigations into the effects associated with the instantaneous reaction of the system, i.e., with the processes of brittle fracture of the material in the region at the crack tip. Attention is given only to stable subcritical crack propagation leading to the critical state, i.e., spontaneous fracture. This case is of special interest in connection with the problem of increasing the thermal stability of refractory materials.

It was assumed that stable crack growth in a brittle material is possible as a result of an increase in the effective surface energy of fracture $\gamma$ with crack extension [4]. Energy $\gamma$ can be increased either by the gradual branching of the crack [5] or by the formation of a zone of microcracking ahead of the front of the propagating crack [6, 7]. Subcritical crack growth leads to deviations from linear elasticity. This phenomenon is of great interest for practice since, according to Gogotsi [8, 9], it increases the thermal stability of the material.
Attempts were made to analyze the effect of microcracking on the crack resistance of materials on the basis of the energy approach [10, 11]. However, the kinetics of development of the zone of microcracking were not taken into account and, consequently, the dependences of crack extension and displacement (strain) on the stress intensity factor were not determined. Analysis can be carried out utilizing a certain degree of analogy with the theory of subcritical crack growth in the microplasticity conditions developed by Cherepanov [12].

We shall assume that the monotonic loading of a material with an initial crack with length \( l_0 \) is accompanied by microcracking in a specific area at the crack tip characterized by the linear dimension \( r \). Regions of the structure in which microcracking takes place can be represented by grain boundaries, for example. It is also assumed that the loading process is described by the stress intensity factor which in the case of microheterogeneous materials is after all a conventional quantity [13, 14].

In this case, the infinitely small increment of crack length \( d\ell \) depends only on the corresponding increment \( dK_I \) and its instantaneous value and also on the prior history of crack propagation [12].

It is also assumed that the microcracking processes can be described by the approximation of continuous densities of noninteracting microcracks. In this case, the density of the microcracks can be described by a statistical distribution function, e.g., by the function of the following type [2]

\[
\rho = \rho_0 \exp\left[-\left(\frac{K_I}{\sigma_0^{1/2}}\right)^m\right],
\]

where \( \rho_0 \) is the density of the sources of microfailure; \( \sigma_0 \) and \( m \) are the distribution parameters.

According to [2], at a specific density of the microcracks, namely at \( \rho/\rho_0 = 0.5 \), the distance between the microcracks decreases to such an extent that the strength of the effect of the interactions between these microcracks and with the main crack becomes very high. After reaching a specific value, the density of the microcracks will increase at a considerably higher rate and the main crack will propagate. Consequently, the effective size of the zone of microcracking can be determined in accordance with Eq. (1) from

\[
r = \alpha_1(m)\left(\frac{K_I}{\sigma_0}\right)^2,
\]

where \( \alpha_1(m) = (-\ln 0.5)^{-2/m} \).

The structure of Eq. (2) is identical with that of the equation which describes the size of the plastic zone in elastoplastic crack propagation. Equation (2) indicates that the length of the zone of microcracking at given \( K_I \) increases with a reduction in the mean strength of the material \( \bar{\sigma} \) which is proportional to \( \sigma_0 \), and with a reduction in the homogeneity of the structure of the material (e.g., \( \alpha_1 \) increases from 1.09 to 1.41 when \( m \) is reduced from 8 to 2).

The energy balance equation for the increase of crack length by \( \delta\ell \) has the form

\[
2\gamma N = \delta A_e + \delta A_c,
\]

where \( \delta A_e \) is the released elastic energy at \( r = \text{const} \); \( \delta A_c \) is the irreversible work of microcracking determined by the increase of \( r \) by \( \delta r \). The value of \( \delta A_e \) is equal to [12]

\[
\delta A_e = \frac{K_I^2}{E} \delta N,
\]