Processes connected with diffusion in heterogeneous solid bodies, such as dispersed mixtures and matrix systems, form the basis of a number of widely used manufacturing techniques (powder metallurgy, production of ceramics and refractories, heat treatment of metals and alloys). However, the mathematical theory of diffusion is only comparatively rarely used for solving relevant practical problems. The reason for this is that, in spite of various simplifying assumptions which are resorted to in solving diffusion equations for heterogeneous solid bodies, the calculations involved are relatively complex and laborious.

To make the mathematical theory of diffusion in heterogeneous media easier to handle in practical application, the authors have performed the most laborious calculations regularly recurring in the solution of individual problems. The results of these calculations are compiled in the table, which lists concentrations calculated from Eqs. (1) and (2) for various values of \( c_0, \xi, \) and \( \tau. \)

\[
c(\xi, \tau) = \frac{c_1 - c_2}{2} \sum_{n=1}^{\infty} \frac{2}{a_n} \left( \sin \frac{\pi \xi}{3} \right)^2 \exp \left( -\frac{\tau^2}{2n^2} \right), \quad \tau \geq 0.10; \tag{1}
\]

\[
c(\xi, \tau) = c_2 + \frac{c_1 - c_2}{2} \left[ \frac{2}{\sqrt{\pi}} \xi \left( e^{-\frac{\xi^2}{2\tau}} - e^{-\frac{(\xi + \xi^2)^2}{2\tau}} \right) + \text{erf} \left( \frac{\xi}{\sqrt{2\tau}} \right) + \text{erf} \left( \frac{\xi - \xi^2}{\sqrt{2\tau}} \right) \right], \quad \tau < 0.10. \tag{2}
\]

Here, \( c \) is the average concentration of component A in a heterogeneous medium; \( c_1 \) and \( c_2 \) the concentrations of this component at the initial instant of time in inclusions and in the matrix, respectively; \( c_0 \) the bulk concentration of including in the initial state; \( a_n \) and \( v_n \) summation constants taken from \([1]\); \( \xi \) and \( \tau \) dimensionless parameters given by the expressions

\[
\xi = \frac{r}{R} c_0^{1/3}, \quad \tau = \frac{D t}{R} c_0^{1/3}, \tag{3}
\]

where \( r \) is a vector radius originating at the center of an inclusion; \( R \) the mean inclusion radius; \( D \) the diffusion coefficient; \( t \) time.

In the calculations, it was assumed that \( c_1 = 1 \) and \( c_2 = 0 \), i.e., it was considered that, at the initial instant of time, component A is wholly concentrated in inclusions and in the matrix, respectively; \( c_0 \) the bulk concentration of including in the initial state; \( a_n \) and \( v_n \) summation constants taken from \([1]\); \( \xi \) and \( \tau \) dimensionless parameters given by the expressions

\[
c = \overline{c} + (c_1 - c_2) (c_0 - \overline{c}) \quad \text{at} \quad \tau \geq 0.10, \tag{4}
\]

\[
c = c_2 + \frac{c_1 - c_2}{2} (c_0 - c_2) \quad \text{at} \quad \tau < 0.10, \tag{5}
\]

where \( c_T \) is the tabulated concentration value for the given values of \( \xi \) and \( \tau \) under the standard starting conditions.

Equations (1) and (2) were derived in \([1]\), where a study was made of diffusion in a two-component heterogeneous body with full mutual component solubility. It was assumed that component A was uniformly dispersed in component B in the form of spherical inclusions with a radius \( R \). Expression (2) describes the diffusion of component A from an inclusion into the surrounding matrix up to the moment when the presence of the neighboring and more distant inclusions imposes a boundary condition on diffusion; this occurs at the initial stage of homogenization of a heterogeneous body, when \( \tau \) and, consequently, \( \tau \) are small.
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Table of Concentrations Calculated from Eq. (1) for $\tau \leq 0.10$ and from Eq. (2) for $\tau \leq 0.10$.  

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