GROWTH OF THE CORROSION LAYER IN THE VICINITY OF A CRACK PROPAGATING AT A CONSTANT SPEED

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The fatigue cracks which propagate in corrosion-active gas media (e.g., in the products of fuel combustion) are surrounded by a corrosion layer whose growth is controlled by diffusion processes [1]. In most cases, the crack has the form of a narrow wedge-shaped channel filled with porous corrosion products (Fig. 1). It can be assumed that the porous matter which filled the channel (crack) is a relatively efficient conductor of the corrosion-active agent and, consequently, the main resistance to the diffusion flow is exhibited by the dense layer of the corrosion products which surround the channel.

We assume that the crack moves in a straight line at a constant speed and the corrosion-active agent travels inside the crack without obstacles. After a certain period of time the processes of growth of the crack and corrosion layer should be in the state of dynamic equilibrium: the crack and the external contour of the corrosion layer travel at the same speed. The determination of this contour is also the aim of this work.

The problem is formulated as follows. A crack whose speed vector has the components \( V_x, V_y \) moves in an infinite two-dimensional solid. The edges of the \( F_0 \) are characterized by the constant concentration of the corrosion-active agent* \( c_0 \); at the external contour of the corrosion layer \( F_1 \) the concentration of this agent is equal to \( c_1 \):

\[
c|F_0 = c_0; \quad c|F_1 = c_1,
\]

where \( c_1 < c_0 \). The chemical reactions which take place at the boundary of the corrosion layer lead to a sudden reduction of the concentration of the active agent by the value \( \Delta c \)

*It is assumed that the corrosion-active agent diffuses. We can also use a reversed assumption and assume that metal atoms or ions diffuse. The course of solution of the problem does not change.
Fig. 1 A corrosion-fatigue crack in a boiler pipe (42-mm diameter, 6-mm wall) made of 12Kh1MF steel.

Fig. 2. Image of the parabolic region with the section at the half axis x < 0 in the plane z under band in the plane w. The numbers denote the characteristic points of the contour and their images.

which is assumed to be given. The diffusion process is assumed to be quasistationary. This means that the concentration c of the active agent within the limits of the corrosion layer must satisfy Laplace's equation

\[
d^2c/dx^2 + d^2c/dy^2 = 0.
\]  

(2)

The matter balance condition must be satisfied at the boundary of the corrosion layer \( \Gamma_1 \) [2]; for two-dimensional diffusion, this condition has the form

\[
D \partial c/\partial n|_{\Gamma_1} = -\Delta c ((\partial \Gamma_x/\partial t)n_x + (\partial \Gamma_y/\partial t)n_y).
\]

D is the diffusion coefficient; \( dc/dn \) is the derivative of the concentration along the normal to the contour; \( \Gamma_x(t) \) and \( \Gamma_y(t) \) are the components of the vector-function of the scalar argument \( t \) which characterizes the displacements of the point of the contour \( \Gamma_1 \) with time; \( n_x, n_y \) are the directing cosines of the external normal to the contour \( \Gamma_1 \).

As already mentioned, in the quasistationary state the contour travels in the same direction and with the same speed as the crack. Consequently, \( d\Gamma_x/dt = V_x; d\Gamma_y/dt = V_y \).

Taking this into account, the condition of the balance of matter can be written in the form

\[
D \partial c/\partial n|_{\Gamma_1} = -\Delta c (V_x n_x + V_y n_y).
\]

(3)

To solve this problem, it is necessary to determine the contour \( \Gamma_1 \) for which the function \( c(x, y) \) is determined in the range restricted by the contour \( \Gamma_0 \) and \( \Gamma_1 \), and satisfies the Eq. (2) and the conditions (1) and (3).

The solution is constructed using the method of conformal images. We shall examine, in particular, the image given by the function

\[
z = w^2/2.
\]

(4)

It is well known [3] that this image transfers the straight lines \( u = \text{const} \) in the plane of the complex variable \( w = u + iv \) into a family of parabolas in the plane \( z = x + iy \). The equation of these parabolas can be written in the form

\[
x = x_s [1 - y^2/(4x_s^2)],
\]

(5)

where \( x_s = u^2/2 \) is the distance from the focus, positioned in the origin of the coordinate, to the tip of the parabolas.