GENERAL ANALYSIS OF STABILITY IN SLOW LIQUID COMBUSTION

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In [1] the stability of a laminar flame front in a vapor evaporating from a liquid surface was examined, with consideration of the effects of gravity and surface tension of the phase boundary. The stabilizing influence of these factors was established. In [2] the stabilizing effects of the force of gravity and liquid viscosity were determined. In [1, 2] the studies were carried out within the framework of hydrodynamic stability using the condition of constancy of the normal mass flux on the hydrodynamic discontinuity [1], which connects the evaporation and combustion fronts. The present study will consider the general problem of hydrodynamic and diffusion-thermal stability (in the sense of [3]) of separated evaporation and combustion fronts without consideration of the interaction between motion of the medium and heat propagation. In the limiting case a characteristic equation is obtained for the hydrodynamic discontinuity joining the evaporation and combustion fronts.

1. The Hydrodynamic Problem. Let the media \( \alpha = 1, 2, 3 \) consist of liquid, heated vapor, and combustion products respectively, with surfaces I, II being the evaporation and combustion fronts (the configuration and coordinate system are shown in Fig. 1). As usual, all media are considered immiscible. Along the fronts there propagate perturbations of the surfaces of the media \( \xi^I, \xi^I \), proportional to \( \exp(iky + Gt) \), where \( k \) is the wave number \( \text{Im}(bk) = 0 \), \( \Omega = -i\omega \) (for stability it is necessary that \( \text{Re}(\Omega) < 0 \)). These perturbations produce changes in the normal mass fluxes on the fronts \( \delta J^I, \delta J^I \) and cause appearance of degenerate-acoustic (a) and entropy-turbulent (r) perturbations throughout the entire media (directions of the perturbations are shown in Fig. 1). Let the dependence of a perturbation of any magnitude \( \phi \) upon \( x \) have the form

\[
\begin{align*}
\psi_{10-} &= \psi_{10-}e^{k(x-x_1)}, & \psi_{10+} &= \psi_{10+}e^{-k(x-x_1)}, \\
\psi_{20-} &= \psi_{20-}e^{-k(x-x_1)}, & \psi_{20+} &= \psi_{20+}e^{k(x-x_1)}, \\
\psi_{30-} &= \psi_{30-}e^{k(x-x_1)}, & \psi_{30+} &= \psi_{30+}e^{-k(x-x_1)}, \\
\psi_{10+} &= \psi_{10+}e^{k(x-x_1)}, & \psi_{10+} &= \psi_{10+}e^{-k(x-x_1)},
\end{align*}
\]

where \( l \) is the wave number \( r \) of the perturbation; \( x_I = -d < 0, x_{II} = 0 \) are the coordinates of the fronts, the primes refer to perturbations, and the symbol \( \Delta \) denotes perturbation amplitudes on the fronts. Perturbations with \( \text{Re}(l_\phi) \neq 0 \) have a physical meaning. From the perturbed continuity and Navier-Stokes equations

\[
\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 0;
\]

we obtain

\[
\rho_a \left( \frac{\partial}{\partial t} + u_a \frac{\partial}{\partial x} - v_a \Delta \right) \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \psi_a = 0,
\]

where \( K_{ar} = \partial u_a/\partial x - \partial u_a/\partial y \) is the z component of the velocity rotor and the other notation is conventional.

In light of Eq. (1.1), we obtain from Eq. (1.3) the following dispersion equations

\[
z_a + w_{ar} = \frac{\lambda_{ar}}{2}(w_a + 1),
\]

where \( z_a = \Omega/(u_a k) ; w_{ar} = L_a/k ; \lambda_{ar} = 2\nu_a k/u_a \). From Eq. (1.2) we have the relationship

On the surfaces of the discontinuities the following boundary conditions are fulfilled:

continuity of mass flow

\[ \delta j_i = \rho_1 \delta u_{i1} = \rho_2 \delta u_{i2}, \quad \delta j_{i1} = \rho_2 \delta u_{i2} = \rho_3 \delta u_{i3}; \]  \hspace{1cm} (1.5)

interrelation of normal momentum components

\[ (\rho'_1 + 2j \delta u_1 - \tau_{i1x})_1 - (\rho'_2 + 2j \delta u_2 - \tau_{i1x})_1 = - \partial \tau_{i1x} / \partial y + (\rho_1 - \rho_2) g_z, \]
\[ (\rho'_2 + 2j \delta u_2 - \tau_{i2x})_{21} - (\rho'_3 + 2j \delta u_3 - \tau_{i2x})_{21} = (\rho_2 - \rho_3) g_z; \]  \hspace{1cm} (1.6)

continuity of tangential momentum components

\[ (j \delta v_{x1} - \tau_{x1})_1 = (j \delta v_{x2} - \tau_{x1})_{21}, \]
\[ (j \delta v_{x2} - \tau_{x2})_{21} = (j \delta v_{x2} - \tau_{x2})_{21}; \]  \hspace{1cm} (1.7)

continuity of tangent stresses

\[ \tau_{x1} = \tau_{x2}, \quad \tau_{x2} = \tau_{x3}; \]  \hspace{1cm} (1.8)

Here \( \sigma \) is the surface tension coefficient and

\[ \tau_{xxx} = 2\eta_a \partial u_a / \partial x, \quad \tau_{asy} = \eta_a (\partial u_a / \partial x + \partial u_a / \partial y). \]