VELOCITY OF FLAME PROPAGATION UPON DEVELOPMENT OF HYDRODYNAMIC INSTABILITY

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On the basis of a theoretical analysis and numerical calculations, it has been shown that the dynamics of a flame surface under conditions of hydrodynamic instability can be represented as the interaction of a finite number of nonlinear configurations of the flame front. Their number is defined by the physical size of the system in which the flame propagates. It has been shown that the development of an initial plane front leads to a stationary regime at which the propagation velocity of the curved flame tends asymptotically to its limiting value independent of the size of the system in which burning takes place. This conclusion is based on the exact solution of the nonlinear equation describing hydrodynamic instability of flame.

It is known that one of the methods for describing the hydrodynamic flame instability is to represent the flame as a surface which propagates along its normal with the velocity $S$ and separates two gases of different density: the fresh mixture and the combustion products. The propagation velocity along the normal depends on the curvature at a given point of the surface. This dependence, which is obtained with approximate account of the thermal structure of the flame front, can be written as [1]

$$S = S_0 \left(1 + \sigma \frac{\partial f}{\partial x} \right).$$

Here $\sigma$ is the Markstein constant proportional to the thermal thickness of the flame, $z = f(x, t)$ is the equation of the flame surface. Let us further consider that $\sigma > 0$, which corresponds to the flame front stability from the standpoint of the theory of diffusive thermal stability [2].

Despite these simplifying assumptions of the flame front structure, the problem is still complicated because the surface curvature causes changes in the velocity and pressure of the fresh mixture and combustion products. In the case of a small gas expansion coefficient $E - 1 = \varepsilon \ll 1$, where $E = \rho_1/\rho_2$ is the density ratio of fresh gas and combustion products, the problem is substantially simplified. In the weakly nonlinear approximation ($\partial f/\partial x \ll 1$), instead of solving a complete system of equations, it is sufficient to consider one equation describing the evolution of the disturbances of the flame surface [3, 4]:

$$\frac{\partial f}{\partial t} + S_0 \frac{x}{2} \{Kf\} - \sigma S_0 \frac{\partial f}{\partial x} - \frac{S_0}{2} \left(\frac{\partial f}{\partial x}\right)^2 = 0,$$  

$$\{Kf\} = \lim_{\varepsilon \to 0} \frac{\varepsilon}{\pi} \int_{z=x}^{z-x} \frac{z}{z^2 + (x - \eta)^2} f(\eta) \, d\eta.$$  

The integral term $\{Kf\}$ can be written as the Hilbert operator with the integral in terms of the main value [5]:

$$\{Kf\} = \frac{1}{\pi} \int_{z=x}^{z-x} \frac{\partial f(\eta)}{\partial \eta} \frac{1}{z-x} \, d\eta.$$  


Equation (1) is written in the coordinate system of the plane flame, which propagates along the axis $z$ with velocity $S_0$, and some of its terms can be obtained from a simple geometrical standpoint. The surface, each elementary part of which moves along its normal with the velocity $S_0$, is described by the equation

$$\frac{df}{dt} - S_0 - V_z = S_0 \left( 1 + \frac{\sigma'_f}{\sigma_x} \right) \sqrt{1 + \left( \frac{\sigma'_f}{\sigma_x} \right)^2}. \quad (2)$$

Here $V_x$ and $V_z$ are the disturbances of the velocity components of the fresh gas, which are due to the curvature of the plane flame front. As shown in [4], the disturbances of the velocity $V_z$ are in the first approximation related to deflections of the flame surface from the plane as follows:

$$V_z = -(K_f) S_0 x / 2.$$

Taking into account this fact and retaining in (2) terms not higher than the second order of smallness in the parameter $|\sigma'_f/\sigma_x| \ll 1$, we can easily derive Eq. (1).

Numerical calculations [1] show that the flame surface has the form of constantly interacting cells: large cells disintegrate, small cells are absorbed by the large ones, etc. However, the calculations mentioned do not answer the question about the time dependence of the propagation velocity of the curved flame and the amplitude of the cells. This is because it is difficult to numerically simulate the evolution of the flame surface over a long period of time. The answer is probably obtained in this study. Henceforth, by the propagation velocity of the curved flame we mean the quantity $c(t)$ (an analog of the turbulent velocity) determined as follows:

$$f(x, t) = c(t) t + p(x, t), \quad p(x, t) < \text{const},$$

where $c$ specifies the velocity of motion of the plane surface, the deviations from which are described by the function $p(x, t)$. In the case of infinite space, such a representation requires the supplementary substantiation that $p$ is bounded. If the size of