Adaptive remeshing for shear band localization problems*

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Summary: Failure of earth structures or laboratory specimens of soils is often characterized by the existence of bands or surfaces at which strain localizes. Numerical simulation of shear band development has attracted the attention of many research groups during past years. Here it is proposed that adaptive remeshing techniques can be applied to better simulate strain localization problems in geotechnique. The algorithm has been previously applied to compressible fluid dynamics problems to capture discontinuities such as shocks. A new refinement functional has been introduced to improve quality of produced meshes. Finally, the algorithm is applied to solve inception and development of shear bands on both homogeneous stress fields and non-homogeneous stress fields.

Adaptive Netzverfeinerung bei Problemen mit Scherfugen-Lokalisierung


1 Introduction

Failure of soil structures and foundations is very frequently accompanied by development of surfaces or bands at which high gradients of strain are present. Precise determination of conditions under which failure takes place has been one of the main concerns of geotechnical engineering. Early efforts were devoted to obtaining failure surfaces and their associated safety factors. Later, finite element techniques allowed a more precise analysis of the stress and strain fields, using more sophisticated material models. However, capture of failure surfaces by standard finite element techniques is still not satisfactory as deformation is smeared over a certain number of elements.

During past years two approaches have been followed to improve the modelling of localized deformation, shear band inception and development.

Bifurcation analysis techniques are based on early work of Hadamard [1], Thomas [2], Hill [3], Mandel [4] and Rice [5]. Basically, localization of deformation is assumed to arise as consequence of material behaviour which determines the shear band direction.

Experimental and theoretical work by Vardoulakis, Goldscheider and Gudehus [6], and Vardoulakis [7] allowed both to understand and predict shear band inclination in biaxial tests.

* Presented at the workshop on Numerical Methods for Localization and Bifurcation of Granular Bodies, held at the Technical University of Gdansk (Poland), September 25—30, 1989
carried on dry sand. Numerical simulation techniques were also proposed by Borst [8, 9] for geomaterials.

Alternatively, a second approach is based on the introduction of inhomogeneities such as soft inclusions, causing very intense concentrations of strain in narrow zones emerging from them. Numerical simulation consists basically on introducing soft or triggering elements, as suggested by Prevost [10], Bardet [11] or Shuttle and Smith [12]. Again, the band is smeared over the elements and a large number of elements is needed to accurately capture the band. Recently, Ortiz et al. [13–15] have proposed a method based on the introduction of enhanced basis functions which are added to the standard solution at bifurcated elements, taking into account band directions.

The approach proposed here is based on an adaptive remeshing algorithm developed by Peraire et al. [16] for compressible flow computations, and applied to produce constant error finite element meshes by Zienkiewicz and Zhu [17]. It was previously shown [18] that this technique was able to provide good resolution for homogeneous field problems in which a soft inclusion had been used. The aim of this paper is to extend past work to non-homogeneous field problems, introducing a refinement function based on bifurcation.

2 Adaptive mesh refinement

Adaptive mesh refinement techniques aim to produce an optimal finite element grid for a given problem for which the exact solution is not known. Optimization will be dependent on the purpose of the analysis. Sometimes, we will be interested in arriving at iso-error grids for elastic problems, and an error estimator will have to be defined [17], while in others such as encountered in high speed, supersonic flow the analyst will concentrate on capturing shocks as accurately as possible [18].

Here, the main interest is the design of finite element grids able to resolve sharp shear bands. We have to remark that location of the band is not known a priori, and therefore it is necessary to begin with an initial mesh which provides a first solution which is used to obtain an improved mesh. After few iterations the position of the shear band is accurately determined. The proposed method consist, therefore, on two steps.

First, the mesh corresponding to iteration \( j \) is used to obtain a solution of the problem. This mesh is characterized by a set of mesh parameters \( \textbf{P}^j \), describing element sizes, aspect ratio, etc. Once the new solution is obtained, it is possible to define a new \( \textbf{P}^{j+1} \) characterizing an improved mesh \( (j + 1) \). Of course, the determination of \( \textbf{P}^{j+1} \) depends on the objective searched by the analyst. Should it be keeping the error constant over the computational domain, the grid size could be obtained using the procedure described in [17]. For compressible flow computations the technique is different, and it aims to produce very fine grid sizes in regions of sharp changes of field variables such as shocks [16].

Here, the problem to be solved is similar, because of the existence of discontinuities in the derivatives of field variables, i.e. in the strain.

For one-dimensional problems the mesh is fully characterized if the spacing \( \delta \) is given at any point of the domain as

\[
\delta^2 \left| \frac{\partial^2 \xi}{\partial x^2} \right| = C_1
\]

where \( \xi \) is a functional of field variables and their derivatives and \( C_1 \) is a constant which defines the degree of refinement. We can conveniently specify the minimum mesh subdivision \( \delta_{\text{min}} \). Then \( C_1 \) can be obtained by applying the condition

\[
\delta_{\text{min}}^2 \left| \frac{\partial^2 \xi}{\partial x^2} \right|_{\text{max}} = C_1
\]

which gives

\[
\delta^2 = \left| \frac{\partial^2 \xi}{\partial x^2} \right|_{\text{max}} \delta_{\text{min}}^2 \left| \frac{\partial^2 \xi}{\partial x^2} \right|_{\text{max}}
\]