CALCULATION OF COMBUSTION REGIMES FOR A TWISTED GAS FLOW
IN A TUBULAR IDEAL DISPLACEMENT REACTOR

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Using the equations of the hydrodynamics of a twisted flow, we have employed a numerical method to construct the temperature and concentration fields of an exothermically reactive gas. We have analyzed the regimes of combustion and the mechanism involved in the ignition of the flow at various values of the twist parameter.

The unique features of combustion within a flow are governed, on the one hand, by the structure of the flow, since the velocity field defines the convective transfer of heat and of the reagents, while on the other hand it is governed by the sensitivity of the reaction rate to the temperature and concentration of reagents, which determine the intrinsic velocity of flame-front distribution through a nonmoving medium.

The simplest fuel-influx scheme in the form of a uniform flow produces a steady combustion wave [1, 2], and the position of the front here is stabilized at some distance from the point of fuel influx, which depends on the relationship between the rate of reagent influx and the intrinsic burning velocity. If combustion occurs in the channel, then the transfer of heat through the walls may lead to flame blow-off, and this will create a low-temperature regime for the conversion of the reagent into the final product. Our examination of the simplest shearing flows, for example Poiseuille flows, introduces no fundamental additional factors to these regimes [3].

If the transfer of heat blocks the formation of a high-temperature burning zone, special measures can be employed to stabilize combustion. For example, a reactor wall temperature may be specified significantly to exceed the original flow temperature, so that the heating of the wall would initiate burning. This overheating may be generated by an excess flow of heat through a good conducting wall out of a section exhibiting maximum flow temperature.

An alternative method may involve the organization of a special type of flow which would increase the stay time of the substance in the reactor, for example the twisting of the flow. Twisted flow through a tube, moreover, intensifies the stirring of the flow and, this is most important, given sufficiently strong twisting the appearance of a recirculation zone becomes possible, the latter offering additional stabilization of the combustion [4].

In the following we examine the problem of steady fields of velocities, concentrations, and temperature in the influx of a twisted reaction-capable flow into a tube whose wall temperature $T_w$ is higher than the temperature $T_0$ of the incoming flow. The area in which the parameters of the chemical reaction undergo change is such in this case that the ignition regime and flame formation within the tube for the case in which $T_w = T_0$ become impossible.
The mathematical model of this flow will be formulated by using the variables of the stream function, i.e., the strength of the vortex [5]. This allows us to eliminate pressure as an explicit variable and automatically to monitor the retention of the quantity of substance downstream. For a viscous incompressible steady flow we will write out the following corresponding equations of hydromechanics:

\[
\begin{align*}
\frac{\text{Re}}{4} \frac{\partial}{\partial x} \left[ \frac{\omega}{2} \frac{\partial \psi}{\partial x} \right] - \frac{\partial}{\partial x} \left[ \frac{\omega}{2} \frac{\partial \psi}{\partial x} \right] - \left\{ \frac{\partial}{\partial x} \left[ \frac{\omega}{2} \frac{\partial \psi}{\partial x} \right] + \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} \right) \right\} - \text{Re} \frac{\partial^2 \psi}{\partial x^2} = 0,
\end{align*}
\]

Here \( x \) is the longitudinal coordinate referred to the tube radius \( r_0 \); \( \xi \) is a dimensionless radial coordinate; \( \psi \) is the dimensionless tangential velocity whose scale becomes clear from the formulation of the boundary conditions. This pertains equally to the stream function \( \psi \) with whose aid the axial and radial components of velocity are reproduced:

\[
\begin{align*}
v_x &= \frac{1}{2 \xi} \frac{\partial \psi}{\partial \xi}, \quad v_\xi = -\frac{1}{2 \xi} \frac{\partial \psi}{\partial x},
\end{align*}
\]

and it pertains equally to the vortex strength

\[
\omega = -\frac{1}{2} \left( \frac{\partial v_x}{\partial \xi} - \frac{\partial v_\xi}{\partial x} \right).
\]

In specifying the boundary conditions we assumed that the distribution of velocity at the inlet to the tube corresponds to the law governing the rotation of a solid:

\[
\begin{align*}
x = 0: \psi = \xi^2, \quad \omega = 0, \quad v_\xi = \sigma \frac{v}{r_0},
\end{align*}
\]

where \( \sigma = \Omega r_0 / U \) is the twist parameter; \( \Omega \) is the angular velocity at the inlet to the reactor; \( U \) is the velocity at which the material flows into the tube. In addition to \( \sigma \), the hydrodynamics are also characterized by the Reynolds number \( \text{Re} = 2U r_0 / \nu \) (\( \nu \) is the coefficient of kinematic viscosity).

"Soft" boundary conditions were imposed at the tube outlet, to simulate the free outflow of the fluid:

\[
\begin{align*}
x = L: \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial \omega}{\partial x} = \frac{\partial v_\xi}{\partial x} = 0.
\end{align*}
\]

Bearing in mind the constancy of the rate of flow and adhesion, at the wall we have

\[
\begin{align*}
\xi = 1: \psi = 1, \quad v_\xi = 0,
\end{align*}
\]

and at the axis of the tube we have

\[
\begin{align*}
\xi = 0: \psi = 0, \quad v_\xi = 0.
\end{align*}
\]

The vortex strength at the axis and at the tube wall was determined by the method from [5].

In writing out the equations for the concentration and temperature fields, we assumed the presence of an exothermic reaction following the first sequence:

\[
\begin{align*}
\frac{\text{Re} \cdot \text{Sc}}{4} \left\{ \frac{\partial}{\partial x} \left[ \eta \frac{\partial \psi}{\partial x} \right] - \frac{\partial}{\partial \xi} \left[ \eta \frac{\partial \psi}{\partial x} \right] \right\} - \left\{ \frac{\partial}{\partial x} \left( \xi \frac{\partial \psi}{\partial \xi} \right) + \frac{\partial}{\partial \xi} \left( \xi \frac{\partial \psi}{\partial \xi} \right) \right\} - \xi Da (1 - \eta) \exp \left( \frac{\theta - \theta_a}{1 + \beta (\theta - \theta_a)} \right) = 0,
\end{align*}
\]

(2)