In the determination of the limiting state of a material an important role is played by the initial stage of its stress-strain diagram, where the elastic and plastic properties of the material are so closely interlinked that it is extremely difficult to distinguish between them, since the transition from elastic to predominantly plastic properties occurs within a very small region. Clearly, this region can only be assessed approximately, using residual strain tolerances.

As an arbitrary yield stress in tensile and compression testing it is usual to adopt the stress at which the residual strain is

$$\delta_t = \delta_e = 0.002 = 0.2\%.$$  \hspace{1cm} (1)

The arbitrary yield stress $\sigma_{0.2}$ can then be regarded as a result which has given rise to a certain plastic strain by any means - simple extension, plastic working, or heat treatment. The main purpose of the tolerance is to enable such conditions to be satisfied which will make it possible to compare certain properties of a material in various stressed states.

The cases of extension and compression are characterized by a uniform distribution of stress within given sections, but in the majority of cases this condition is not satisfied, e.g., in bending and torsion. In bending it is possible in principle to apply the tolerance (1) to the strain of the layers most distant from the neutral plane. In torsion, however, the problem is more complex. Here a solution may be provided by determination, on the basis of theory, of an equally effective (equivalent) strain. R. Ludwig [1], starting from considerations of equivalence of stresses at various magnitudes of $\sigma_{\text{max}}$, derived the expression

$$\delta_{\text{tor}} = 2\delta_t = 0.004 = 0.4\%.$$  \hspace{1cm} (2)

N. N. Davidenkov, pointing to an error in R. Ludwig's approach, obtained the following expression for the torsional tolerance [1]:

$$\delta_{\text{tor}} = 1.5\delta_t = 0.003 = 0.3\%.$$  \hspace{1cm} (3)

A torsional tolerance in the form of Eq. (3) is not the only possible one. Using, e.g., a hypothesis of equality of octahedral tangential stresses, we get

$$\delta_{\text{tor}} = 0.0035 = 0.35\%.$$  \hspace{1cm} (4)

Consequently, the choice of a residual strain tolerance in the determination of the yield stress of a material depends to a large extent on the criterion of equivalence of strains. Development of a method of yield stress determination is of fundamental importance, from both a theoretical and a practical point of view, in the mechanics of nonsolid media. For porous solids, the following condition may serve as a criterion of equivalence of strains:

$$J_0 = 3J_2' + \alpha J_1^2 - \beta \sigma_y^2 = 0;$$  \hspace{1cm} (5)

$$\tilde{J}_0 = \frac{3}{2} S_{ij} S_{ij} + \alpha \sigma_{hk} \sigma_{ij} - \beta \sigma_y^2 = 0.$$  \hspace{1cm} (6)

Using a flow law associated with the condition (5) or (6), we obtain
\[
\delta_{\text{eq}} = \frac{3}{2} \frac{d \varepsilon_{\text{eq}}}{\sigma_{\text{eq}}} \left[ \sigma_{ij} - \frac{1 - 2\alpha}{3} \sigma_{hh} \sigma_{hh} \right] \quad (i, j = 1, 2, 3).
\]

Let us consider the case of linear extension

\[
\sigma_1 = \sigma_0^\text{y}, \quad \sigma_2 = \sigma_3 = 0.
\]

where \( \sigma_0^\text{y} \) is the yield stress in tension. The tolerance then will be

\[
\delta_t = \frac{3}{2} \frac{d \varepsilon_{\text{eq}}}{\sigma_{\text{eq}}} \left[ \sigma_1 - (1 - 2\alpha) \sigma_1 \right]
\]

or, allowing for Eq. (7),

\[
\delta_t = \frac{d \varepsilon_{\text{eq}}}{\sigma_{\text{eq}}} (1 + \alpha) \sigma_0^\text{y}.
\]

In shear (torsion), when \( \sigma_1 = -\sigma_3 \) and \( \sigma_2 = 0 \), the residual strain tolerance may be expressed as

\[
2\delta_\gamma = d \varepsilon_{\text{eq}} - d \varepsilon_{\text{eq}} = 3 \frac{d \varepsilon_{\text{eq}}}{\sigma_{\text{eq}}} (\sigma_1 - \sigma_3),
\]

or

\[
2\delta_\gamma = 3 \frac{d \varepsilon_{\text{eq}}}{\sigma_{\text{eq}}} 2\tau_\gamma.
\]

With Eq. (8), the shear tolerance may be written

\[
2\delta_\gamma = 3 \frac{\delta_t}{1 + \alpha} \frac{\tau_\gamma}{\sigma_0^\text{y}}
\]

or

\[
\delta_\gamma = 3 \frac{\delta_t}{2} \frac{\tau_\gamma}{1 + \alpha} \frac{\sigma_0^\text{y}}{\sigma_0^\text{y}}.
\]

Thus, the shear tolerance has been found to be dependent on the ratio between the yield stresses in shear and tension, which can be established from the limiting state condition

\[
S_\mu + \sigma_0^2 = \beta \sigma_0^2.
\]

Hence, bearing in mind that in shear \( \sigma_0 = 0 \), we find

\[
\tau_\gamma / \sigma_0^\text{y} = \sqrt{\beta}.
\]

Thus, the tolerance in shear is linked with the tolerance in tension by the expression

\[
\delta_\gamma = \frac{3}{2} \frac{\delta_t}{1 + \alpha} \sqrt{\beta}.
\]

Turning to solid metals \( (\alpha = 0 \text{ and } \beta = 1) \), we arrive at the relationship established by N. N. Davidenkov,

\[
\delta_\gamma = 1.5\delta_c.
\]

Consequently, this approach to the determination of tolerance is a natural generalization of the method adopted for solid metals and alloys. In the case of porous metals, residual strain tolerances depend on the nonsolidity parameter, since the functions \( \alpha \) and \( \beta \) depend on porosity: \( \alpha = \alpha(\varnothing) \) and \( \beta = \beta(\varnothing) \). In a similar manner it is possible to establish the relationship between the tolerances in omnidirectional uniform compression and shear or tension,

\[
\delta e_\mu = 9 \alpha \frac{d \varepsilon_{\text{eq}}}{\sigma_{\text{eq}}} P.
\]

Allowing for Eq. (11), we can express the relationship between the tolerances in shear and isostatic compression in the form

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