Lag-Zone Calculation Scheme

As has been demonstrated earlier, the theory of loose (particulate) materials can be employed for the analysis of stresses within metal powder in the zone of rising compaction pressure in the rolling mill (the lag zone) [1]. In this zone, the powder is in an unstable, critical state. The shear strain is so large that part of the powder is pushed into higher layers of the lag zone [1, 2]. Bearing in mind the small thickness of this zone, it may be assumed that the condition of limiting equilibrium establishes itself throughout its whole thickness. Under limiting equilibrium conditions, rupture of a loose material takes place along slip and discontinuity surfaces.

For metal powder, as for the majority of loose materials, above a certain relatively low value the magnitude of the shear strain has a negligible effect on the limiting equilibrium, including the value of the internal friction angle \( \rho \). For this reason, in an analysis of stresses and slip and discontinuity lines, the lag zone with rotating rolls may be replaced by a scheme where the shear strains, although small, exceed the limit at which they affect the limiting equilibrium.

As the roll length is appreciable, the influence of the end walls may be ignored, so that the problem to be solved becomes a plane one.

With the speeds at which rolling is nowadays performed, the inertia forces acting on the powder do not exceed the forces due to its weight. If this were not the case, with powder flowing in under gravity, conditions would be established favoring the formation of voids within its volume. This is not observed in powder rolling practice.

We will assume that the hopper unit sets up at the boundary, during the simultaneous replenishment of the lag zone with powder, some uniformly distributed pressure \( P \) which is sufficient to enable us to neglect the weight and inertia forces of the powder present in the lag zone; thus, the medium is taken to be weightless. The roll surfaces act as supporting walls. Boundary conditions show that, in relation to the load (pressure \( P \)), the loose medium is in a maximally stressed state.

The stressed state of the loose medium (normal and tangential stresses \( \sigma_x, \sigma_y, \) and \( \tau_{xy} \)) will be described in terms of the reduced stress \( \alpha \) and the angle between the principal normal stress and the x axis (angle \( \phi \)). The changeover to the principal normal stresses \( \sigma_1 \) and \( \sigma_2 \) for the loose medium involves the relations:

\[
\sigma_1 = \sigma(1 + \sin \phi), \quad \sigma_2 = \sigma(1 - \sin \phi). \tag{1}
\]

Let us introduce the auxiliary variables \( \xi \) and \( \eta \), which are related to \( \sigma \) and \( \phi \) by the following expressions:

\[
\xi - \eta = 2\phi, \quad \xi + \eta = \ln \frac{\sigma}{\sigma_0} \tan \phi. \tag{2}
\]

where \( \sigma_0 = \text{const} \). It may be noted that \( \xi = \text{const} \) for the slip curves of the first family and \( \eta = \text{const} \) for the slip curves of the second family.
A diagram for the calculation of the lag zone is shown in Fig. 1. The calculation of the lag zone for a weightless loose medium is reduced to solving two equilibrium equations for an elementary volume of powder [3]

\[ \sum X = 0; \quad \frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} = 0, \quad \sum Y = 0; \quad \frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} = 0 \]

and the limiting equilibrium equation \( \sigma_1/\sigma_2 = K \) [3].

In solving the problem represented by the calculation scheme adopted, we employ a procedure similar to that used for a loose medium bounded by two supporting walls [3]: For the part of the lag zone bounded by the slip or discontinuity line O'DE (ODE'), we obtain a precise analytical solution; a solution for the remaining part (zone VI) is found by an approximate method, using recurring formulas (Fig. 1).

Let us consider the part of the lag zone bounded by the line O'DE (ODE'). Depending on the thickness of the hopper unit and, consequently, the feed angle \( \alpha_0 \), three types of stressed state may be distinguished: