A study of the Bailey–Orowan equation of creep

DERSHIN GAN
Brookhaven National Laboratory, Upton, New York 11973, USA

A new method was developed to study the Bailey–Orowan equation of creep, $\dot{\varepsilon}_c = r/h$, where $\dot{\varepsilon}_c$ is the creep rate, $r$ is the recovery rate and $h$ is the work-hardening coefficient. The method was to vary the strain rate, $\dot{\varepsilon}$, around the creep rate, $\dot{\varepsilon}_c$, and to measure the corresponding stress rate, $\dot{\sigma}$. In a plot of stress rate against strain rate, a straight line was obtained. The slope of the straight line was equal to $h$, and the intersection of the straight line with the stress axis was equal to $-r$, as in the equation $\dot{\sigma} = -r + h\dot{\varepsilon}$. The creep test under a constant stress is a special case of this equation when the stress rate, $\dot{\sigma}$, is zero. The above measurement was carried out within a very small stress variation, less than 1% of the total stress, so that the values of $r$ and $h$ were not disturbed. The creep test was performed on Type 316 stainless steel. The creep rate was shown to be equal to the ratio $r/h$, but the value of $h$ was approximately equal to Young’s modulus at the testing temperature, rather than, as is commonly believed, to the work-hardening coefficient.

1. Introduction

It was first suggested by Bailey [1] that the creep at elevated temperature is a process in which the work hardening from deformation is continually annealed-out by recovery. Steady-state creep occurs when a balance between work hardening and recovery is reached. A simple mathematical formulation of the recovery theory was proposed by assuming that the stress, $\sigma$, is a function of time and strain, $\varepsilon$:

$$d\sigma = \left(\frac{\partial \sigma}{\partial t}\right)_\varepsilon dt + \left(\frac{\partial \sigma}{\partial \varepsilon}\right)_t d\varepsilon = -r dt + h d\varepsilon,$$

(1)

where $r$ is the recovery rate and $h$ is the work-hardening coefficient. Since the stress is constant in a creep test, $d\sigma = 0$ and Equation 1 becomes

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon}_c = r/h,$$

(2)

where $\dot{\varepsilon}_c$ is the creep rate. This is the Bailey–Orowan [2] equation of creep.

Though this concept of creep is generally accepted, experimental verification has proved elusive. In earlier work [3–6], $r$ was determined by the stress transient dip test in which a stress reduction, $\Delta\sigma$, (about 10% of the total stress) was applied during the creep test and then the incubation time $\Delta t$ was measured to determine the recommencement of creep. The value of $r$ was approximated by the ratio $\Delta\sigma/\Delta t$. Similarly, a stress jump, $\Delta\sigma$, could be applied during the creep test and the corresponding strain increment, $\Delta\varepsilon$, was then measured. The value of $h$ could be approximated by $\Delta\sigma/\Delta\varepsilon$ [4–6]. These experiments generally gave an order-of-magnitude agreement between the value of $r/h$ and the measured steady-state creep rate. However, this method of measuring $r$ and $h$ was seriously questioned by Lloyd and McElroy [7–9], who presented evidence that the observed incubation time in the stress dip test was a consequence of anelasticity. Anelasticity could also not be ruled out in the stress–jump test. The measurements of $r$ and $h$ are therefore dubious.

In this work a new method has been developed to study the Bailey–Orowan equation. This method is based on the following equation derived from Equation 1 by dividing each term by $dt$,

$$\dot{\sigma} = -r + h\dot{\varepsilon},$$

(3)

where $\dot{\sigma}$ is the stress rate $d\sigma/dt$ and $\dot{\varepsilon}$ is the strain rate $d\varepsilon/dt$. Since creep tests are conducted under constant stress, the Bailey–Orowan equation is
only a special case of Equation 3 when the stress rate \( \dot{\sigma} \) is zero and the strain rate, \( \dot{\varepsilon} \), is equal to the creep rate, \( \varepsilon_c \). Experimentally, the strain rate \( \dot{\varepsilon} \) was varied around the creep rate, \( \varepsilon_c \), and the corresponding stress rate, \( \dot{\sigma} \), was recorded. A straight line was obtained in a plot of stress rate against strain rate and, according to Equation 3, the slope of the straight line should be equal to \( h \) and the intersection of the straight line with the stress axis should be equal to \( -r \). The measurements of the stress rate and the strain rate variations were taken within a very small stress variation, less than 1% of the total stress, so that the values of \( r \) and \( h \) were not disturbed. Besides the verification of Equation 2, the dependence of \( r \) and \( h \) on creep strain, temperature and stress was also studied in this experiment.

2. Experimental procedure

Creep specimens were machined from a 2.54 cm diameter rod of Type 316 stainless steel. The composition of the steel is 0.06 wt % C, 1.57 wt % Mn, 0.029 wt % P, 0.023 wt % S, 0.75 wt % Si, 16.8 wt % Cr, 10.7 wt % Ni and 2.16 wt % Mo. The specimen geometry is shown in Fig. 1. All specimens were annealed in air at 1050 ~ C for 30 min and then air-cooled with a resultant average grain size of 60 \( \mu \)m. Specimens were creep tested in a mechanically driven model 1125 Instron Universal Testing Machine. This machine is ideal for this experiment because of a special “hold” control designed to hold the specimen at a constant load. A load limit can be selected by a ten-turn potentiometer in the range of the load cell. When the increasing load reaches the load limit during the cross-head upward movement, the hold control stops the cross-head and allows the specimen to relax under a fixed cross-head. A less than 1% decrease of the total load reactivates the “up” control, and the cross-head starts moving up again at the selected speed. The load is again increased until it reaches the load limit and triggers the hold control. By repeating this process, the sample is subjected to a stepped creep test with a cyclic load amplitude of less than 1% of the total stress. To begin a test, the cross-head was first brought up at high speed until the load limit was reached. Then it was changed to low speed, generally \( 8.33 \times 10^{-7} \) m sec\(^{-1} \) to complete the creep test. This stepped creep test is actually equivalent to a real creep test conducted under constant load. The analysis will be shown later.

We shall denote the strain rate of the specimen in the loading period by \( \dot{\varepsilon}_l \), and that in the holding period by \( \dot{\varepsilon}_h \). The corresponding stress rates are denoted by \( \dot{\sigma}_l \) and \( \dot{\sigma}_h \), respectively. Obviously \( \dot{\sigma}_l \) must be positive while \( \dot{\sigma}_h \) is negative because the load increases in the loading period but decreases in the holding period. The strain rate of the specimen in the loading period, \( \dot{\varepsilon}_l \), is directly controlled by the cross-head speed. So different strain rates \( \dot{\varepsilon}_l \) can be obtained simply by choosing different cross-head speeds. The cross-head speed used for normal creep tests is \( 8.33 \times 10^{-7} \) m sec\(^{-1} \), but when \( r \) and \( h \) are to be measured, the cross-head speed is varied over a wide range of speeds to obtain different strain rates, \( \dot{\varepsilon}_l \), and stress rates, \( \dot{\sigma}_l \). This kind of measurement can be performed at any stage of creep. The variable cross-head speed unit allows the choice of any other cross-head speeds, in addition to those controlled by standard push buttons.

Creep tests were conducted in air at 750 ~ C under a constant load limit with an initial stress of 103.4 MPa. The temperature was maintained within \( \pm 2 \) ~ C throughout the length of the specimen by a seven-zone furnace with a temperature controller. The extension of the specimen was measured by means of an extensometer attached to the shoulders of the specimen. An Instron strain

![Figure 1](image-url)