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ANALYSIS OF WEIGHT OPTIMIZATION OF SPIRAL-ANNULAR-WOUND CYLINDRICAL SHELLS WITH LAYERS HAVING DIFFERENT PHYSICOMECHANICAL CHARACTERISTICS*

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The possibility of controlling the anisotropy of a packet of layers permits the design of laminated structures which will best satisfy the requirements established for them. By efficient, or optimal, we mean a structure which under the influence of external loads \( q \) and in the presence of constraints functions in the best manner, i.e., the structure is characterized by a set of parameters \( x_0 \) for which the objective function \( \Phi(x) \) takes an extreme value:

\[
\Phi(x_0) = \text{extr } \Phi(x).
\]

The system of constraints is introduced by means of the vector function \( \varphi(x) \):

\[
\varphi(x) \leq 0,
\]

where \( x \) is the set of parameters characterizing the shell. These parameters may be the angles of winding of the layers, their elastic and strength characteristics, or other structural characteristics of the shell. Thus, the optimum design \( x_0 \) is the set of shell parameters that ensures the minimum of the objective function \( \Phi(x) \). The weight of the

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structure \( Q(x) \) is often used as the objective function, with restrictions on load-carrying capacity and deformation. It is convenient to use the following weight optimization parameter

\[
W(x) = \frac{p_x V}{G},
\]

where \( p_x \) is the breaking pressure; \( V \) is the internal volume of the shell; \( G \) is the weight. As will be shown below, \( W \) is expressed through the unit strengths of the material of the shell layers. The optimum shell design \( x_0 \) for a pressurized shell was found from the equation

\[
W(x_0) = \max_x W(x).
\]

For several reasons — mainly because of the difficulties of finding the extremum of the objective function — instead of by solving Eq. (1) we find the optimum design \( x_0 \) from simpler conditions such as equal strength, equal stress, equal weight, etc. Here, it is understood that these conditions are also conditions of the extremum of the objective function. Such an approach was examined in [1-13]. Several other studies have in essence asserted but not proven that the optimality conditions which are introduced make it possible to obtain a design of minimum weight. In all of the studies [1-13], the optimality conditions formulated in them are reduced to certain relations between components of the stress–strain state without the use of strength criteria of the material or the failure mechanism of the structure. Given this approach, it is impossible to establish a relationship between the geometric parameters of the shell and the physiomechanical characteristics of the layers, including strength characteristics, that will permit realization of the design. In other words, optimizing structures only through relations linking components of the stress–strain state — without the use of strength criteria and the failure mechanisms of the structure — makes it impossible to unambiguously connect the limiting loads of the optimum shell with the strength of its layers. Proving the extremity of the objective function \( Q(x) \) or \( W(x) \) is also problematic with this approach.

The authors of [1] formulated optimality conditions in the form of the requirement that the stress state at all points of the structure belong to the strength surface of the layer packet. Here, it is assumed that the material of the packet is a homogeneous anisotropic medium. Such an approach can be used to solve optimization problems only when the strength criterion for the packet is known for any set of parameters. In [2], the optimality criterion was formulated in the form of the principle of equal strength — the requirement that failure occur at all points of the structure simultaneously. The conditions formulated in [2] are rather general and can be reduced to equal-stress conditions only for the simplest structures made of a homogeneous material. Satisfaction of these conditions will evidently make it possible to obtain a structure of the minimum weight. However, although this was postulated in [1, 2], it was not proven.

In the general case of structures made of an inhomogeneous material or of materials with different mechanical characteristics, it is very difficult to analytically describe the condition of simultaneous failure, and this condition may differ appreciably from the