DETERMINATION OF THE SURFACE-FINISH PARAMETERS
BY MEANS OF THE TOTAL INTERNAL REFLECTION
METHOD

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The application of the Mechatl method (disruption of total internal reflection) for determining the actual contact area between the surface of the investigated body and the glass prism is described in detail in [1, 2]. In this method the measured surface is pressed against the glass prism. In the contact areas the total internal reflection of light is disrupted, thus producing dark spots on a light background. However, this method is unsuitable for evaluating the tested surface-roughness parameters for materials with a large modulus of elasticity (steel, glass, ceramics, etc.), since in the case of such materials contacts with the glass prism occur, even for large pressures, only at separate most protruding irregularities. Moreover, such irregularities are subjected during contacting to plastic deformations, thus disrupting their original shape.

Below we suggest a modification of the Mechatl method, which is suitable for evaluating the quality parameters ($R_a$ and $H_{10}$) of the measured surface. The contact surface of the measuring prism is covered with a transparent rubbery film (see Fig. 1). The modulus of elasticity $E$ of this film is approximately 10$^4$ times smaller than that of glass. Therefore, even at insignificant pressures, the contact area of the measured surface with the film is large (it amounts to tens of percent of the total surface area) and the surface remains virtually without deformations. As a result of this it becomes possible to determine the quality parameters of the tested surface by solving the corresponding contact problems of the theory of elasticity. The contact area is measured by means of the photodetector.

Below we derive formulas for evaluating the quality parameters of certain widely-used types of machined surfaces.

Let the surface be represented by a field consisting of cylindrical waves. The vertical cross section of this surface can be represented by a Gaussian random stationary process with the mean value of 0. Let us determine the pressure on an elastic half space of an irregular surface immersed into the space to a depth of $h$ with respect to the zero level. The distribution function of the irregularity heights will then be equal to $\Phi(h/\sigma)$.

If the height is sufficiently large, it is possible to assume with considerable trustworthiness that the spikes have the shape of a parabola and the duration $x$.

If the parabola's derivative at the point of intersection is $y$, the parabola's height will be $b = xy/4$ and the second derivative at the vertex will be $2y/x$. The parabola's curvature at the vertex will also be equal to $2y/x$ and the radius of curvature to

$$R = \frac{x}{2y}.$$  \hspace{1cm} (1)

A single parabolical cylinder of unit length is subjected along its generating line to the force $F$. The formula for determining $F$ is provided in [3] as

$$F = \frac{3\pi x^2}{16RD},$$  \hspace{1cm} (2)

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Fig. 1

where

\[ D = \frac{3}{4} \left( \frac{1-v^2}{E} + \frac{1-v'^2}{E'} \right) \]

is the modulus of elasticity, \( \nu \) is the Poisson coefficient.

In our case the value of \( (1-\nu^2)/E \) can be neglected for the material under consideration, since the modulus of elasticity \( E' \) for rubber is considerably smaller.

It will be seen from (1) and (2) that

\[ F = \frac{3\pi xy}{8D} = \frac{3\pi b}{2D}. \] (3)

The value of the mean force per unit length can now be easily determined as

\[ \bar{F}(h) = n_{av}(h) \cdot \int_{h}^{b} \frac{3\pi (b-h)}{2D} \rho(b, h) \, db. \] (4)

Here \( n_{av}(h) \) is the mean number of spikes per unit length, determined from the formula

\[ n_{av}(h) = \frac{1 - \Phi \left( \frac{h}{\sigma} \right)}{q(x, h) \, dx}, \] (5)

where \( q(x, h) \) is the distribution of the spike lengths at the level \( h \); \( \rho(b, h) \) is the distribution density of spike heights at the level \( h \).

It is shown in [4] that in many instances the distribution function of irregularity heights can be adequately approximated by means of the Rayleigh density

\[ \rho(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}. \] (6)

Let us determine for this case the relationship of the contact area to the pressing force and the surface-roughness parameter \( \sigma \).

The nominal distribution density of the maxima lying above the level \( h \) is determined from the formula

\[ p(x, h) = \rho(x) e^{-\frac{x^2}{2\sigma^2}}. \] (7)

It is shown in [4] that the duration of spikes at high positive levels also has a Rayleigh distribution with the density of

\[ q \left( \frac{h}{\sigma} \right) = - \frac{1}{4} R_0^* \frac{h^2}{\sigma^2} \tau \exp \left( \frac{1}{8} R_{0}^{\nu} \frac{h^2}{\sigma^2} \tau^2 \right). \] (8)

Therefore,

\[ \int_{0}^{\infty} q \left( \frac{x}{\sigma} \frac{h}{\sigma} \right) \, dx = \frac{\sqrt{\pi} \sigma}{2 \sqrt{-R_{0}^{\nu} h}}. \] (9)