NUMBER OF LEVELS OF INSTRUMENT CHECKING CHARTS

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Checking charts indicate the order in which the accepted units are transferred from standards to working instruments [1]. These charts establish the metrological hierarchy of reference measuring instruments, their accuracy at each checking level, and the principles of instrument comparison.

In spite of the great importance of checking charts in metrology their theoretical foundations have not been fully explored so that the design of checking charts is based largely on the experience and intuition of the specialist.

The problem of the number of levels of a checking chart and of the relative accuracy of reference measuring instruments on adjacent levels has been treated in [2, 3] from the point of view of minimizing the cost of the checking net. Obviously, this requires the knowledge of the dependence of equipment and labor cost on accuracy. Lack of such information led the authors to conclude that "... optimum checking charts cannot be designed in practice" [3].

Such an analysis is nevertheless of considerable importance as it can reveal the effect of different factors on the cost of a checking net. In particular, it has been noted that the function describing the dependence of the cost on the distribution of errors in the reference instrument levels and on the number of such levels has a quite broad minimum.

Here we propose a technique for determining the number of levels of reference measuring instruments on the basis of the available initial data.

In practice, the selection of the number of reference instrument levels is based on many different considerations. One has to take into account the range of the measured physical quantity, the gradation of the working instrument accuracy, the abundance of such instruments and their checking periods, the branching of the checking net, the accuracy of reference measuring instruments, the efficiency of checking operations, etc. Obviously, the calculation of the number of necessary checking levels which would allow for all these factors is hardly realistic.

To solve this problem theoretically it is necessary first of all to isolate the most relevant characteristics of checking charts.

As such we will take the overall margin of accuracy of the chart, i.e., the ratio of the maximum permissible errors of working measuring instruments to the error of the standard, and the efficiency of the checking net.

Denote the maximum permissible errors of working instruments by $\delta_m$ and the maximum error of the standard by $\delta_s$. Their ratio $Q = \delta_m / \delta_s$ defines the overall margin of accuracy of the checking chart.

Let us write $Q$ as the product of the ratios of maximum errors of measuring instruments on adjacent levels of the chart:

$$Q = \prod_{i=1}^{m} q_i,$$

where $q_i = \delta_j / \delta_{j-1}$.

Here $q_1 = \delta_1 / \delta_s$ and $q_m = \delta_m / \delta_{m-1}$. The number of levels of reference measuring instruments is $m - 1$.

Because of the natural error accumulation, always $\delta_{j+1} > \delta_j$. Frequently, $q_j$ increases with $j$. Translated from Izmeritel'naya Tekhnika, No. 9, pp. 8-9, September, 1972. Original article submitted May 14, 1971.

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Reference instruments on the upper levels of the checking charts are usually tested by calibration and certification. The ratio $q_j$ depends on the process of error accumulation \[4\] whose analysis makes it possible to find the minimum value $q_{\text{min}}$.

In the existing checking charts \[1\] $q_{\text{min}} = 1.2 - 1.4$

Checking of working measuring instruments (and sometimes also of reference instruments) is usually limited to testing if their errors do not exceed some specified limits. In this case $q_i$ can be found from the specified probability that the test fails \[5, 6\]. This gives $q_{\text{max}}$.

It has been found in practice that most frequently $q_{\text{max}} = q_{\text{m}}$ and reaches up to 10.

If $q_{\text{min}}$ and $q_{\text{max}}$ are known, we can find approximately the number of levels $m - 1$. Applying the mean-value theorem we get for $m$

$$
\left( \frac{1}{m} \sum_{j=1}^{m} q_j \right)^m > \prod_{j=1}^{m} q_j.
$$

Figure 1

Considering (1), we have

$$
\log Q / \log q,
$$

where

$$
q = \frac{1}{m} \sum_{j=1}^{m} q_j \quad \text{and} \quad \bar{q} = \left( \prod_{j=1}^{m} q_j \right)^{1/m}.
$$

As an estimate of $\bar{q}$ let us take the geometric mean

$$
\bar{q} = (q_{\text{min}} \cdot q_{\text{max}})^{1/m}.
$$

Let us leave in (2) the sign of equality and replace $\bar{q}$ by its estimate $\bar{q}$:

$$
\hat{m} = \log Q / \log \bar{q}.
$$

Rounding off $\hat{m}$ to the nearest integer $m^*$ we find the number of levels $m^* - 1$.

In particular, if $q_j = \text{const} = q_{\text{min}}$, the maximum possible number of levels is

$$(m - 1)_{\text{max}} = (\log Q / \log q_{\text{min}}) - 1.$$

Assume that we know the time $t_{(j+1)}$ necessary for checking the measuring instruments on level $(j + 1)$ and the time $T_{0j}$ for which the reference instrument of level $(j)$ can be used between check-up periods of the tested instruments.

The number of reference instruments on level $(j + 1)$ covered by the check-up is given by

$$
N_{(j+1)} = N_j \frac{T_{0j}}{t_{(j+1)}}.
$$

Considering successively the entire checking chart, from the standard down to working instruments, we have

$$
N_m = \prod_{j=0}^{m} \frac{T_{0j}}{t_{(j+1)}},
$$

where $j = 0, 1, ..., m$, the subscript $j = 0$ denoting the standard.

In most fields of measurement working instruments have different accuracies and ranges. To allow for this, checking charts provide branch-offs. The checking chart has then a structure shown in Fig. 1.

In the presence of branches, the efficiency of a checking chart must be evaluated for every branch separately.