A factorization method is used to solve a group of electrodynamic problems involving the radiation of dipoles at a straight discontinuity in the surface impedance specified on an infinite plane and at the edge of an impedance halfplane tangential to these systems.

A factorization method was used in [1] to solve the problem of the radiation of a point source at the boundary of semiinfinite planar systems—planes having straight discontinuities in the surface impedance or halfplanes with equal surface impedances in an elastic medium. In this paper the method is applied to the electromagnetic analogs of the same problem: the case in which the radiation source is an elementary magnetic or electric dipole in the plane of the system (the version in which the dipole is perpendicular to the plane of the system requires a slightly different approach; it was treated elsewhere [2]) and whose axis either coincides with the boundary of the plane (case x) or is perpendicular to this boundary (case y).

We first consider an infinite impedance plane $z = 0$ with a straight cut $\Delta Z = z_p - z_n$ in the surface impedance $Z$ ($Z = z_n$ for $y < 0$ and $Z = z_p$ for $y > 0$) which coincides with the x axis of a Cartesian coordinate system $Oxyz$. We assume that at point O there is a magnetic dipole $(m)$ in either the x or y orientation (we denote these cases by $x_m$ and $y_m$, respectively). The total field in the region $z > 0$, which may be described by a magnetic Hertz vector $\Pi (\Pi_x, \Pi_y, 0) = \Pi (H_x, \Pi_y)$ tangential to the plane of the system (i.e., a two-component vector), must satisfy the boundary condition $[zE] = Z [z\Pi]$ on the plane $z = 0$. With an account of the field representation chosen, this condition may be rewritten as

$$\begin{align*}
(z_p L - i\kappa \pm \partial / \partial z) \Pi &= 0, \quad y < 0; \\
(z_p L - i\kappa \pm \partial / \partial z) \Pi &= 0, \quad y > 0; \quad z = 0,
\end{align*}
$$

(1)

where $L$ is a linear differential operator with the matrix

$$L = \begin{pmatrix} \kappa^2 + \partial_x^2 & \partial_x \partial_y \\ \partial_x \partial_y & \kappa^2 + \partial_y^2 \end{pmatrix}$$

and $\varepsilon$ is a unit operator.

We represent the unknown vector as an integral superposition of plane waves:

$$\Pi = \frac{i}{(2\pi)^2} \int \pi(z) e^{i\kappa z} d z,$$

(2)

in which $\pi(\gamma)$ is a new unknown vector, a function of the vector variable $\gamma(u, v)$ such that $d \gamma = du dv$. In addition, $\rho$ is a vector with the components $x, y$; and $K = (\kappa_x^2 - \kappa_y^2)^{1/2}, \text{Im} K \geq 0$. Applying a double Fourier transformation to (1) with an account of (2), we find the following system of functional equations for the transform $\pi$:

$$(z_p T + \kappa K) \pi = M^-, (z_p T + \kappa K) \pi = M^+.$$

(3)

Here $T$ is an operator with the matrix

$$\begin{pmatrix} \kappa^2 + \partial_x^2 & \partial_x \partial_y \\ \partial_x \partial_y & \kappa^2 + \partial_y^2 \end{pmatrix}$$

and $\mathbf{M}^\pm = \mathbf{M}^\mp (\nu)$ are two-component vectors $[\mathbf{M} = \mathbf{M}(\mathbf{M}_x, \mathbf{M}_y)]$, regular in the lower and upper half-planes, respectively, of the complex $\nu$ plane and having a common regularity band for $\text{Im} \, \kappa > 0$ containing the real axis.

As in [3, 4], we find the operator $D$ with the matrix

$$D = \begin{pmatrix} -u & \nu \\ u & -\nu \end{pmatrix},$$

which diagonalizes matrix $\mathbf{T}$, converting it to the form

$$\mathbf{R} = \mathbf{D} \mathbf{T} \mathbf{D}^{-1} = \begin{pmatrix} \kappa^2 & 0 \\ 0 & \kappa^2 \end{pmatrix},$$

this operator thus transforms system (3) into a set of independent equations

$$(z_n \mathbf{R} + \kappa \mathbf{K} s) = \mathbf{N}^-, \quad (z_p \mathbf{R} + \kappa \mathbf{K} s) = \mathbf{N}^+$$

for the new unknown vector $s$. Here $\mathbf{N} = \mathbf{D} \mathbf{M} (\mathbf{N} = \mathbf{N}(N_1, N_2))$. Eliminating the vector $s$ with an account of (4), and "factorizing" (writing in product form) the functions

$$K + \kappa \mathbf{Z} = \overline{\mathbf{K}} + \overline{\kappa} \overline{\mathbf{Z}} = \overline{\mathbf{Z}} \overline{\mathbf{K}} \overline{\kappa},$$

$$\kappa + K \mathbf{Z} = Z(K + \kappa Y) = Z(\overline{\mathbf{K}} + \overline{\kappa} \overline{\mathbf{Y}}) = Z \overline{\mathbf{Z}} \overline{\mathbf{K}} \overline{\kappa},$$

$$\mathbf{N}^{-} = N^{-} = P_1(\nu), \quad \mathbf{N}^{+} = N^{+} = P_2(\nu),$$

(see [3, 4]), we find the functional equations

$$N_1^{\pm} \frac{\dot{\nu}}{\dot{\nu}_n} = N_1^{\pm} \frac{\dot{\nu}}{\dot{\nu}_p} = P_1(\nu), \quad \sqrt{\frac{z_p}{z_n}} N_1^{\pm} \frac{\dot{\nu}}{\dot{\nu}_n} = \sqrt{\frac{z_n}{z_p}} N_1^{\pm} \frac{\dot{\nu}}{\dot{\nu}_p} = P_2(\nu),$$

(5)

where $P_1(\nu)$ and $P_2(\nu)$ are certain polynomials in integral powers of the variable $\nu$.

Having determined the vector $\mathbf{N}$ from (5), and omitting the intermediate calculations, we find the vector $\pi$ from $\pi = A^{-1} \mathbf{D}^{-1} \mathbf{N}$, where $A = Z \mathbf{T} + \kappa \mathbf{K} \mathbf{e}$. Substituting this expression for $\pi$ into (2), we find the general solution for the problem formulated:

$$\Pi_x = \frac{i}{(2\pi)^2} \int \left[ -\frac{\nu P_1(\nu)}{\kappa \dot{\nu}_n} \frac{\dot{\nu}}{\dot{\nu}_p} + \frac{\nu P_1(\nu)}{V \dot{\nu}_n} \frac{\dot{\nu}}{\dot{\nu}_p} \right] e^{i \nu_0 + i \kappa z} \frac{dx}{x^2},$$

$$\Pi_y = \frac{i}{(2\pi)^2} \int \left[ \frac{\nu P_1(\nu)}{\kappa \dot{\nu}_n} \frac{\dot{\nu}}{\dot{\nu}_p} + \frac{\nu P_1(\nu)}{V \dot{\nu}_n} \frac{\dot{\nu}}{\dot{\nu}_p} \right] e^{i \nu_0 + i \kappa z} \frac{dx}{x^2}.$$