The present paper is a refinement and elaboration of the ideas of [1] on the applicability of the Feynman procedure in the nonlinear spinor theory of elementary particles. The possibilities of probabilistic interpretation in a scheme with an indefinite metric and of constructing a unitary physical scattering matrix are analyzed. The modified perturbation theory proposed in [1] is used to compute meson masses and meson-nucleon coupling constants. The results turn out to be physically satisfactory.

In [1] we cited considerations in favor of the applicability of a somewhat modified perturbation theory in the Heisenberg variant of nonlinear spinor theory [2] and used this theory to compute the nucleon mass for various types of self-action [sic]. However, we did not on that occasion discuss the problem of probabilistic interpretation and of constructing a unitary physical scattering matrix. We shall attempt to fill this gap here, using the axiomatic approach for brevity and clarity. Self-evident postulates such as those of relativistic invariance and spectral character are omitted. The remaining postulates are not to be regarded as absolutely rigid: some of them may turn out to be consequences of others, and some may require more rigorous mathematical formulation. At the end of the paper the modified perturbation theory is used to obtain more specific results, namely to compute meson masses and meson-nucleon coupling constants.

Axiom I. The space of states $H$ in nonlinear spinor theory carries an indefinite metric (IM). Its vectors $|\varphi\rangle$ have asymptotic limits, and the condition of completeness of the space of asymptotic states is fulfilled, i.e.,

$$H_{\text{in}} = H_{\text{out}} = H. \tag{1}$$

Since we have chosen the Heisenberg representation, the asymptotic states are to be understood in the sense of Ruelle [3] or Hepp [4], i.e., their existence actually presupposes the existence of field operators. But the latter will be discussed below.

We introduce the IM in order to improve the behavior of the casual function and to circumvent the implications of the Kallen-Lehmann representation. The fact that the metric is indefinite means that the physical vectors of state $|\Phi\rangle \langle\Phi| \in H_{\text{in}} \subset H$ with the positive norm $\langle\Phi|\Phi\rangle \geq 0$ (equality for $|\Phi\rangle = 0$ only) do not constitute a complete system in $H$. They must be complemented by nonphysical ("ghost") states $|\varphi\rangle$ for which $\langle\varphi|\varphi\rangle \leq 0$. Thus, $H = H_{\text{in}} \oplus H_{\text{out}}$, $H_{\text{in}} \perp H_{\text{out}}$, or

$$|\Psi\rangle = \left( \begin{array}{c} |\Phi\rangle \\ |\varphi\rangle \end{array} \right) = |\Phi\rangle + |\varphi\rangle,$$

$$\langle\Psi|\Psi\rangle = \langle\Phi|\Phi\rangle + \langle\varphi|\varphi\rangle,$$

$$|\Phi\rangle = P|\Psi\rangle \in H_{\text{in}}; \quad |\varphi\rangle = (1 - P)|\Psi\rangle \in H_{\text{out}}. \tag{2}$$

Here $P$ is the projection operator of $H$ on $H_{\text{in}}$. As usual, it is linear, Hermitian (with respect to scalar multiplication in $H$), and has either the property $P^2 = P$ or $P(1 - P) = 0$. From now on we assume that $H$ is separable.

We introduce the assumption of the existence of asymptotic states in order to allow ourselves to describe the states of free (in- and out-) particles in the usual way. Hypothesis (1) is in a certain sense equivalent to the adiabatic hypothesis (see [1]) and is necessary if we wish to make use of perturbation theory.

Axiom II. There exists a local field operator $\Psi(x)$ which satisfies the local commutativity condition. The vacuum vector $|0\rangle$ is cyclic relative to this operator. The field operator has the asymptotic limits $\Psi_{\text{in}}(x)$ and $\Psi_{\text{out}}(x)$.

This axiom gives a more precise version of the properties of the operator $\Psi(x)$.

Strictly speaking, one should specify $\Psi$ in the form of unbounded generalized operator functions and stipulate their domain of definition very rigorously. But this is not important for the discussion to follow, as we shall be dealing with $\Psi$ as though they were ordinary quantities.

Local commutativity is a reflection (though a weak one) of the causality principle. The requirement of a cyclic vacuum enables us to construct the space of states from this vector in the usual way, i.e., by operating on it with polynomial combinations consisting of $\Psi_{\text{in}}$ and $\Psi_{\text{out}}$.

We shall not discuss the meaning of the asymptotic limiting process, since the (far from trivial) problem...
of existence of the corresponding limits does not concern us. We need merely assume the existence of the operators $\Psi_{in}^{(out)}$ and the cyclic character of the vacuum relative to these operators. This allows us to construct $H_{in}$ and $H_{out}$.

Fig. 4

The quantity $\Psi_{in}^{(out)}$ can be resolved into its physical and imaginary parts:

$$\Psi_{in}^{(m)} = P \Psi_{in} P; \Psi_{in}^{(g)} = \Psi_{in} - P \Psi_{in} P. \tag{3}$$

We assume that the 1-particle state generated by the operator $\Psi_{in}^{(m)}$ has the mass $m$; the nature of the corresponding ghost state will be stated more precisely below. The vacuum is cyclic relative to $\Psi_{in}^{(m)}$ in $H_1$ and relative to $\Psi_{in}^{(g)}$ in $H_2$.

**Axiom III.** There exists a unitary operator $S$ which maps $H_{in}$ onto $H_{out}$ and satisfies the condition

$$S^+ P S = P. \tag{4}$$

In Heisenberg's own variant of the theory [2] instead of (4) we have the requirement

$$P S P = S P. \tag{4a}$$

If we wish to use the IM in $H$ without any restrictions and at the same time attempt to preserve the diagram technique, we must define the $S$-matrix by the relation

$$|\Psi_{out}\rangle = S |\Psi_{in}\rangle, S = T \exp \{i \int L^0 (x) dx\}. \tag{5}$$

By virtue of the assumed Hermitian character (with respect to scalar multiplication in $H$) of the Lagrangian, the operator $S$ is unitary, i.e.,

$$\langle \Psi_{out} | \Psi_{out} \rangle = \langle \Psi_{in} | S^+ S |\Psi_{in} \rangle =$$

$$= \langle \Psi_{in} | \Psi_{in} \rangle \quad \text{or} \quad S^+ S = I. \tag{6}$$

We call this the property of "trivial unitary character". However, this approach gives rise to certain obvious difficulties as regards interpretation of states with a negative norm and of negative-probability transitions. Moreover, it entails a clear violation of microcausality (the introduction of the IM is equivalent to the prevalence of effective nonlocal character [5]), and in many cases of macrocausality as well [6]. There have been many attempts to overcome these difficulties (see, e.g., [7-10]). The most plausible of these is Heisenberg's view, which is in fact equivalent to condition (4a). The latter means that if an in-state belongs to $H_1$, then interaction results in its transition to an out-state which also belongs to $H_1$. This approach preserves the probabilistic interpretation, and (as is easy to show) ensures the unitary character of the physical scattering submatrix $PSP$. Unfortunately, it has not yet been proved that relation (4a) is consistent with the remaining postulates.

We have replaced Heisenberg's condition by requirement (4). Let us investigate its meaning. We assume that only those vectors $|\Psi_{in}\rangle$ which have some nontrivial projection on $H_2$,

$$|\varphi_{in}\rangle = (1 - P) |\Psi_{in}\rangle \neq 0, \tag{7}$$

are permissible. Here $|\varphi_{in}\rangle$ is chosen in such a way that its norm is equal to the norm of the vector $|\varphi_{out}\rangle = = (1 - P) S |\Psi_{in}\rangle$.

$$\langle \varphi_{out} | \varphi_{out} \rangle = \langle \varphi_{in} | \varphi_{in} \rangle \quad \text{or} \quad$$

$$= \langle \Psi_{in} | S^+ (1 - P) S |\Psi_{in} \rangle = \langle \Psi_{in} | (1 - P) |\Psi_{in} \rangle. \tag{8}$$

Recalling the trivial unitary character of (6), we immediately arrive at (4). Preservation of the norm of the physical projections is self-evident.

Thus, in our case the ghost states behave as "standing waves" (in the terminology of [9]), and are not manifested in physical processes. The entire space $H$ breaks down into two invariant (in the sense of norm preservation) subspaces with respect to the action of the operator $S$, i.e., the latter is reducible (separable) into a physical and a nonphysical scattering matrix. Each of these matrices is unitary in the ordinary sense.

**Axiom IV.** Asymptotic vectors of state having the same projection on $H_1$ describe the same physical state. This postulate weakens Heisenberg's requirement that $|\Psi_{in}\rangle$ belong to $H_1$, and constitutes an approximation in idea to the Gupta-Bleiler method of introducing an IM in quantization of the electromagnetic field. The naturalness of the axiom follows from the above analysis: quantities of the type $\langle \Phi \mid \Phi \rangle$, $\langle \Phi \mid P \Phi \rangle$, etc., are preserved, as is necessary in a physically meaningful theory.

We must now consider matrix transition elements of the following form:

$$\langle \Phi_{in} | \Phi_{out} \rangle = \langle \Psi_{in} | P S |\Psi_{in} \rangle. \tag{9}$$

The role of the physical matrix is played by the operator

$$\tilde{S} = P S, \tag{10}$$

which is unitary,

$$\tilde{S}^+ \tilde{S} = P \tag{11}$$

by virtue of condition (4). Thus, the problem reduces to finding the ghost vectors $|\varphi_{in}\rangle$ which satisfy relation (8).

**Axiom V.** All ghost states in $H$ occur only as dipoles of zero mass,

$$|\varphi_{in}\rangle = |g_{in}\rangle. \tag{12}$$

The fact is that Heisenberg ([2], p. 6) proved within the framework of the Lee model that the naked constants in the Hamiltonian can be chosen in such a way that all of the ghost states have zero mass and can be combined with a zero-mass physical state into a dipole $|g\rangle$. The latter is to be understood as follows:

$$|g\rangle = \lim_{\Delta E \to 0} \frac{|\Delta \Psi\rangle}{\Delta E};$$