We show that the relativistic equation of the Brownian motion of a classical particle near a trajectory, stable in Lyapunov's sense, is identical with the Klein-Gordon equation. The conditions which make motions of this type possible are connected with cosmology.

If one takes into account the interactions of elementary particles with a fluctuating electromagnetic field, it becomes possible to explain certain observed phenomena, in particular, the Lamb shift of electron levels in the atom and the additional magnetic moment of the electron [1]. In a similar way the analysis of the equations describing the interaction of a classical electron with virtual photons leads to the conclusion that under such conditions its behavior acquires quantum mechanical properties [2]. In general, it was noticed long ago that an electron which can be described by quantum mechanical equations resembles a Brownian particle, i.e. an object, interacting with the medium [3]. De Broglie's "thermodynamics of an isolated particle" is also well known [4]. On the other hand, it becomes obvious that essentially new results concerning the consequences of the effect of the medium (vacuum) upon the behavior of a classical electron enable one to construct a theory of the stability of motion. Before the fundamental discovery made by Andronov in the theory of vibrations—identifying auto-oscillations as Poincaré stable limiting cycles [5]—it was considered as obvious that the phase space of a dynamic system has no distinguishable trajectories. In reality its topological structure may be arbitrarily complex, and auto-oscillations may be connected in a natural way with quantization [6].

Apparently, the most effective way of establishing the connection between the theory of nonlinear oscillations and quantum mechanics consists in making use of the condition of stability of conservative systems discovered by Chetaev [7],

$$\frac{\partial^2 S'}{\partial x^2} + \frac{\partial^2 S'}{\partial y^2} + \frac{\partial^2 S'}{\partial z^2} = 0.$$  (1)

where $S'$ is the solution of the Hamilton-Jacobi equation

$$-\frac{\partial S'}{\partial t} + \frac{1}{2m} \left[ \left( \frac{\partial S'}{\partial x} \right)^2 + \left( \frac{\partial S'}{\partial y} \right)^2 + \left( \frac{\partial S'}{\partial z} \right)^2 \right] + V = 0.$$  (2)

It is easily seen that (1) leads to solutions of the de Broglie wave type. The substitution

$$\varphi = e^{iS'/\hbar},$$  (3)

enables us to write (2) in the following form:

$$i\hbar \frac{1}{\varphi} \frac{d \varphi}{dt} = -\frac{\hbar^2}{2m} \left[ \left( \frac{1}{\varphi} \frac{\partial \varphi}{\partial x} \right)^2 + \left( \frac{1}{\varphi} \frac{\partial \varphi}{\partial y} \right)^2 + \left( \frac{1}{\varphi} \frac{\partial \varphi}{\partial z} \right)^2 \right] + V.$$  (4)

Using (1) we have

$$\left( \frac{1}{\varphi} \frac{\partial \varphi}{\partial x} \right)^2 + \left( \frac{1}{\varphi} \frac{\partial \varphi}{\partial y} \right)^2 + \left( \frac{1}{\varphi} \frac{\partial \varphi}{\partial z} \right)^2 = \frac{1}{\varphi} \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{\varphi} \frac{\partial^2 \varphi}{\partial y^2} + \frac{1}{\varphi} \frac{\partial^2 \varphi}{\partial z^2},$$

and (4) takes the form of Schrödinger's equation

$$i\hbar \frac{1}{\varphi} \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m} \left[ \frac{1}{\varphi} \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{\varphi} \frac{\partial^2 \varphi}{\partial y^2} + \frac{1}{\varphi} \frac{\partial^2 \varphi}{\partial z^2} \right] + V.$$  (5)

This is well known [8]. However, solutions of Schrödinger's equation

$$i\hbar \frac{1}{\psi} \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left[ \frac{1}{\psi} \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{\psi} \frac{\partial^2 \psi}{\partial y^2} + \frac{1}{\psi} \frac{\partial^2 \psi}{\partial z^2} \right] + V$$  (6)

are sought in the form

$$\psi = e^{iS'/\hbar},$$  (7)

where $S$, unlike $S'$, is a complex quantity. We can assume therefore that $S'$ is the real part of $S$,

$$S = S' + iS';$$  (8)

$$\varphi = Re,$$  (9)

where $R$ is a real quantity, i.e., (1) determines only the phase of the variable $\psi = R \varphi$, and for the amplitude of $R$ (or for $\rho = R^2$) some other condition is necessary. It is natural to assume that it should be the diffusion equation

$$\frac{h}{\rho} \frac{1}{\rho} \frac{\partial \rho}{\partial t} = \frac{\hbar^2}{2m} \left[ \frac{1}{\rho} \frac{\partial^2 \rho}{\partial x^2} + \frac{1}{\rho} \frac{\partial^2 \rho}{\partial y^2} + \frac{1}{\rho} \frac{\partial^2 \rho}{\partial z^2} \right],$$  (10)

where the diffusion coefficient is supposed to be equal, as usual, to $\hbar/2m$ [3].
If one bears in mind that
\[
\frac{1}{\gamma} \frac{\partial \gamma}{\partial t} = \frac{1}{2} \left( \frac{1}{\gamma} \frac{\partial \gamma}{\partial t} - \frac{\partial \gamma^2}{\partial \gamma} \right)
\]
and also that
\[
\frac{1}{\gamma} \frac{\partial \gamma}{\partial t} = \frac{1}{2} \left( \frac{1}{\gamma} \frac{\partial \gamma}{\partial t} - \frac{\partial \gamma^2}{\partial \gamma} \right),
\]
from the left, we get the equations
\[
\frac{1}{\gamma} \frac{\partial \gamma}{\partial t} + \frac{\gamma}{\partial x} \frac{\partial \frac{\partial \gamma}{\partial x}}{\partial \gamma} + \frac{i \gamma}{\partial z} \frac{\partial \frac{\partial \gamma}{\partial z}}{\partial \gamma} = \frac{mc}{\hbar} \gamma - 0.
\]
from (16) is to (4). In the same way, one can obtain from (14) the equations
\[
\frac{1}{\gamma} \frac{\partial \gamma}{\partial t} + \frac{\gamma}{\partial x} \frac{\partial \frac{\partial \gamma}{\partial x}}{\partial \gamma} + \frac{i \gamma}{\partial z} \frac{\partial \frac{\partial \gamma}{\partial z}}{\partial \gamma} = \frac{mc}{\hbar} \gamma - 0.
\]
which are related to (10) in the same manner as equation (16) is related to (5). From (12) one can deduce the Klein-Gordon equation
\[
\frac{1}{c^2} \frac{\partial^2 \gamma}{\partial t^2} - \frac{\partial^2 \gamma}{\partial x^2} - \frac{\partial^2 \gamma}{\partial y^2} - \frac{\partial^2 \gamma}{\partial z^2} + \frac{mc^2}{\hbar^2} \gamma = 0.
\]
Thus, is (16) really is a relativistic equation, describing stable motions, and (18) is the relativistic diffusion equation, the Brownian motion of a classical particle near a trajectory, stable in Lyapunov's sense, is described by the quantum mechanical equation. We want to emphasize the point that both stable motions and the Brownian motion are characteristic only for nonautonomous systems. It is impossible to obtain conditions (13) and (14) for a classical elementary particle in Minkowski space, which does not interact with a vacuum.

In the theory of gravitation it seems to be quite attractive to attempt to show that the metric of space far from the place of mass concentration is not something of primary significance and may be deduced by analysis of gravitational interaction of the systems with the universe at large. Because electromagnetic interaction is much stronger than the gravitational one, it is natural to assume that the electromagnetic field, the presence of which may explain the quantum mechanical properties of particles, is formed by the totality of all particles in the universe.

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