CAVITY MEASUREMENT OF THE PARAMETERS OF A MAGNETIZED FERRITE

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A description is given of an apparatus for measuring the components of the tensor for the permeability and also the dielectric constant. Results are given for Mg-Cr-Cu ferrite. The application to the static magnetization is discussed.

Introduction

The UHF parameters of \( \varepsilon \) (dielectric constant) and \( \| \mu \| \) (permeability tensor) appear in the wave equations for a waveguide or resonant cavity containing a ferrite. Development reports on ferrites give quantities such as the magnetization, density and specific resistance, which are related to the above by the Polder-Hogan equations \([1, 2]\) for ferrites magnetized to saturation. Rado \([3]\) has derived relations for lower degrees of magnetization. Both sets of relations agree satisfactorily with experiment only for a restricted part of the centimeter range and for the millimeter range \([4]\); there is only general agreement at longer wavelengths. In addition, these relations often fail to give satisfactory agreement with experiment for most devices employing polycrystalline ferrites. The reason is that Polder's and Rado's relations neglect effects arising from the anisotropy, grain structure, porosity, and other major physical characteristics. The actual UHF parameters have therefore to be measured.

There are many papers \([5-9]\) on the measurement of \( \varepsilon \) and \( \| \mu \| \), most of them being based on an approximate perturbation treatment; only two \([8, 9]\) employ rigorous solutions. We have used the method of \([9]\) with a cylindrical cavity excited by circularly polarized waves. Such a cavity has a higher Q than the rectangular cavity of \([8]\), and so gives higher sensitivity in loss measurements. The effective scalar permeability may be used for circularly polarized fields in such a cavity, and this gives four of the six components of \( \| \mu \| \). In addition, TM_{nm0} modes give the most complete separation of the electric and magnetic effects in this type of cavity.

Formulas

We derive \( \varepsilon, \mu, \) and \( \mu_a \) from formulas derived from the characteristic equation for a cylindrical cavity having a ferrite rod at the axis and excited in a \( \text{TM}_{nm0} \) mode \([9]\). This equation takes the form

\[
M \frac{\beta a J_n'(\beta a)}{J_n(\beta a)} \pm nK = \frac{ka}{\nu_0} \frac{J_n'(ka) N_n(kb) - N_n'(ka) J_n(kb)}{J_n(ka) N_n'(kb) - N_n(ka) J_n'(kb)},
\]

in which \( M = \frac{v^2}{v_a^2} \), \( K = -\frac{v_a^2}{v^2 - v_a^2} \), \( \mu \) and \( \mu_a \) being the diagonal and nondiagonal components of \( \| \mu \| \); \( J_n(x) \) and \( N_n(x) \) are Bessel and Neumann functions respectively; \( k = \omega \sqrt{\varepsilon_0 \mu_0} \) is the wave number for the part of the cavity outside the ferrite, \( \beta = k \sqrt{\frac{\varepsilon}{\varepsilon_0 \mu_0}} \) is the same for the interior of the ferrite; \( a \) and \( b \) are the radii of rod and cylinder, respectively; and \( n \) is an integer. The + and − signs refer to the two modes of circular polarization.

1. Derivation of \( \varepsilon \): This is based on (1), with \( n = 0, m = 1 \). The dielectric measurements are made with the ferrite unmagnetized, so we put \( \mu_a = 0 \). The perturbation formula may be applied for \( \varepsilon \) not too large if the diameter of the rod is much less than that of the cavity, which for the \( \text{TM}_{200} \) mode is

\[
\varepsilon = 1 + 0.539 \frac{b^2 \Delta f}{a^2 f_0},
\]

in which \( \Delta f \) is the shift in the resonant frequency \( f_0 \) on inserting the specimen.

2. Derivation of \( \mu \) and \( \mu_a \): Here, we put \( n = 1, m = 1 \) in (1) and use the method of \([9]\).

We expand \( \frac{\beta a J_n'(\beta a)}{J_n(\beta a)} \) as a power series in \( \beta a \)

\[
\frac{\beta a J_n'(\beta a)}{J_n(\beta a)} = 1 - \frac{(\beta a)^2}{4} - \frac{(\beta a)^4}{96} - \cdots,
\]

substitute this into (1), and separate the real and imaginary parts to get

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\[ M' + K' = T'_\pm + a^2 \left[ \varepsilon' \left( \omega^2 - \omega_0^2 \right) + 2\varepsilon'\omega'\omega'' \right] \pm \frac{1}{4c^2} + \cdots \]

\[ M'' + K'' = T''_\pm + a^2 \left[ \varepsilon'' \left( \omega^2 - \omega_0^2 \right) - 2\varepsilon'\omega'\omega'' \right] \pm \frac{1}{4c^2} + \cdots \]

in which \( \omega = \omega' - i\omega'' \) is the complex angular frequency of the loaded cavity, while \( T_\pm \) is the right side of (1) multiplied by \( \mu_0, c \) being the velocity of light. \( M \) and \( K \) are found from (4) by successive approximation starting from the value given by perturbation theory:

\[
\frac{\Delta f_\pm}{f_0} = -3.08 \cdot \frac{\mu + \mu_0 - 1}{\mu + \mu_0 + 1} \cdot \frac{a^2}{b^2},
\]

and so on; formula (5) is derived from (1) by taking only the first term in (3).

3. Derivation of \( \mu_2 \): This parallel component is derived from perturbation-theory formulas for a rectangular cavity excited in a \( TE_{102} \) mode:

\[
\frac{\Delta f}{f} = -\frac{\nu_2 - 1}{1 + \left( \frac{L}{2d} \right)^2} \cdot \frac{V_s}{V_c}.
\]

The ferrite rod of sections 1 and 2 was used here also. The additional symbols are: \( V_s \) volume of ferrite, \( V_c \) cavity volume, \( l \) length of cavity, and \( d \) width of wide side. The rod is set parallel to the wide sides equidistant from the ends and sides in an antinode of the UHF field.

**Apparatus and Methods**

Figure 1 shows the block diagram. The klystron 1 with sawtooth frequency modulation provided by generator 3 supplies the cavities 8 and 9, waveguide switch 14 in position a directing the power to cavity 8, and in position b to cavity 9. Cavity 8 is excited with circularly polarized waves of either sense via identical guides leading to two slots 90° apart on the curved surface. Each guide has an attenuator 16 and phase shifter 17. The oscilloscope 7 receives its sweep voltage from unit 5; the unmagnetized ferrite gives rise to a resonance curve on the screen. The Helmholtz coils 20 produce an external magnetic field along the axis, and this gives rise to a second peak; the degeneracy is removed. The two peaks overlap if the rod accounts for only a small fraction of the volume, and if the external field is only a few Oe, because the ferrite simply couples the two circularly polarized modes. Either of these peaks may be suppressed by adjusting the phase shifter 17, which suppresses one of the modes. The attenuators 16 are adjusted to equalize the amplitudes in the two guides. Analogous operations are performed to produce the other sense of circular polarization.

An additional waveguide system is used to measure the parameters of cavity 8 in each condition as well as the parameters of cavity 9. This consists of isolators 11, a klystron 2 (frequency stabilized), a heterodyne wavemeter 6, a crystal mixer 12, tuned amplifier 4, adjustable attenuators 15, and summing unit 5.

Mixer 12 receives inputs from klystrons 1 and 2, the output going to the bandpass amplifier 4, whose central frequency is adjustable from 0.25 to 5 Mc. Attenuators 15 serve to adjust the output from the mixer to the level at which the output contains a negligible proportion of the harmonics of the central frequency of filter 4. The frequency of 1 equals that of 2 twice during a cycle, so the output of 4 peaks twice, the width of the peaks being less than 40 kc (governed by the Q of the filter). The frequency separation of the peaks is twice the central frequency of 4, so the peaks act as calibration marks, which are added to the sawtooth from 3 in the summing unit 5, whose output is the sweep of the oscilloscope 7.

The ferrite isolators 10 serve to decouple klystrons 1 and 2 as well as the other units. These isolators have the following characteristics in their 10% bands; forward loss not more than 1 dB, reverse loss not less than 37 dB; SWR not above 1.1.

The degenerate modes used in cavity 8 demand a highly symmetrical cavity. For this reason, cavity 8 has four coupling holes at intervals of 90° on its surface. The first two are used in excitation; the third is coupled to the detector 19; and the fourth is taken to the matched load 21. Cavities 8 and 9 were made with high precision, of diameters 91.6