ANALYSIS OF A REACTIVE MODULATING AMPLIFIER WITH A SEMICONDUCTING DIODE CAPACITOR

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A modulating amplifier with a p-n transition semiconducting diode capacitor is analyzed by the methods of the theory of oscillations in linear and nonlinear approximations. In the nonlinear approximation the critical mode is calculated. Estimates are made comparing various forms of modulation for effective use in amplification.

At the present time, reactive modulating amplifiers are widely in use. The circuit of such an amplifier can be divided into two parts—the modulator and the detector. The modulator contains a source of alternating emf (the pump) and a nonlinear resistance, the magnitude of which depends on the input signal. Because of the change in the resistance, a signal-modulated pump current flows in the modulator circuit. The detector then separates out the voltage which is similar to the input signal. Under certain definite conditions such a system can give an amplification greater than unity.

Examples of reactive modulating amplifiers are the long-familiar and widely-used magnetic and dielectric amplifiers [1]. Recently, articles have appeared in the literature on amplifiers whose basic element is an oscillating circuit with a nonlinear semiconducting diode capacitor tuned to the frequency of the pump generator [2–6]. Amplification in such a system is accomplished by modulating the pump oscillation with a signal applied to the nonlinear diode capacitor, and subsequently detecting or separating out one of the side frequencies in the oscillation spectrum. Because of their principle of operation these amplifiers also belong to the class of reactive modulating amplifiers. The basic advantage of a reactive modulating amplifier with a semiconducting diode is the possibility of obtaining wide-band amplification with a very low noise level.

Amplifiers of the modulating type have much in common with parametric amplifiers; they contain a nonlinear reactive element and a pump generator, and under modulation a frequency transformation takes place. However, there is one essential point in which they differ from parametric amplifiers, namely that the input to the modulator consists not of a single combination of frequencies, but of an entire modulating spectrum. Consequently, as is noted by Kharkevich [7], the amplification coefficient of a reactive modulating amplifier cannot be determined in a general form on the basis of the Menly–Row equations, but must be determined by analysis of each given specific system.

Analysis of the operation of such an amplifier is carried out by the method of complex amplitudes, which is unable to reveal all the phenomena which arise in the amplification process. In particular, as Withe [8] points out, the amplification in a modulating amplifier involves amplitude, phase, and frequency modulation. Using the method of complex amplitudes, it is not always possible to determine definitely which form the modulation takes for a given tuning of the circuit, and it is also impossible to analyze the nonlinear phenomena which vitally affect the amplifier operation.

We can make up for the above deficiencies by solving a differential equation which describes the operation of the amplifier. In the present article we shall discuss a half-wave amplifier circuit.

1. STATEMENT OF THE PROBLEM

Let us consider a half-wave amplifier circuit with a nonlinear p-n capacitor (Fig. 1). This represents an oscillating circuit with inductance L and the capacitance of a semiconducting diode, to which a reverse bias $E_0$ is applied. Two generators are included in the circuit: one is the pump generator ($E \cos \nu t$), the other is the signal source ($U \cos \Omega t$), while $r$ is the loss resistance of the circuit. Let us assume the conditions

$$\Omega \ll \nu, \quad U \ll E. \quad (1)$$

From [6], in the case of a sharp p-n transition the equation of the oscillating circuit can be written in the form

$$\frac{d^2x}{dt^2} + 2 \lambda \frac{dx}{dt} + \omega^2 x = - \frac{\omega^2 x_0^2}{2} \cdot$$

$$+ \frac{\omega^2}{2} \mu_1 \cos \nu t + \frac{\omega^2}{2} \mu_2 \cos \Omega t, \quad (2)$$

where

$$\omega^2 = \frac{1}{LC_0}, \quad \lambda = \frac{r}{2L}, \quad \mu_1 = \frac{E}{U_0}, \quad \mu_2 = \frac{U}{U_0},$$

$U_0 = E_0 + \varphi K$, $C_0$ is the diode capacitance at the operating point for a small signal, and $\varphi K$ is the diffusion potential. The voltage across the diode is ex-
pressed in terms of the unknown function of time by the relation

$$U_g = U_0 (1 + x)^2. \tag{3}$$

Making the change of variable $x = y + \omega^2 t / (2 (\omega^2 - \Omega^2)) \cos \Omega t$ and using the inequalities (1), Eq. (2) becomes

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + \omega^2 \left(1 + \frac{\mu_1}{2} \cos \Omega t\right) y =$$

$$= - \frac{\omega^2 y^2}{2} + \frac{\omega^2 \mu_1}{2} \cos \Omega t, \tag{4}$$

where $\mu_2 = \mu_1 \omega^2 / (\omega^2 - \Omega^2)$.

Equation (4) describes the forced oscillation of a nonlinear vibrator with a "slow small deviation of the resonance frequency from its original position. This equation may be solved using a method developed by Mitropol'skii [9] for systems with slowly changing parameters. According to this method, we seek a solution of Eq. (4) in the form of a power series in the small parameter $\xi$, and the coefficients of this series are found as functions of amplitude and phase by the method described in [9]. The amplitude and phase of the oscillation, in turn, are determined from a system of reduced equations in a first, second, or higher approximation, depending on the required accuracy.

2. THE SOLUTION OF EQ. (4) IN THE FIRST APPROXIMATION

Using the given method, we obtain in the first approximation $y = a \cos (\omega t + \phi)$, where $a$ and $\phi$ must be determined from the system of equations

$$\frac{da}{d\xi} = - \delta a - \frac{\omega^2 \mu_1}{2 (\omega + \xi)} \sin \phi, \tag{5}$$

$$\frac{d\phi}{d\xi} = \omega - \xi + \frac{\omega^2 \mu_2}{4} \cos \Omega t - \frac{\omega^2 \mu_1}{2 (\omega + \xi)} \cos \phi.$$

When $\mu_2 = 0$ the stationary mode of the oscillation is given by

$$a_0 = \frac{\mu_1}{4 \sqrt{\left(\frac{\omega - \xi}{\omega}\right)^2 + \frac{\delta^2}{\omega^2}}}, \tag{6}$$

$$\phi_0 = \arctan \frac{\omega \mu_1}{\delta \omega}, \quad \delta = \arctan \frac{\omega \mu_1}{\delta \omega}. \tag{7}$$

The presence of a small signal $\mu_2 (\mu_1 \gg \mu_2)$ causes the amplitude and phase of the oscillation to experience small deviations, with frequency $\Omega$, from their stationary values. We may thus use the method of perturbations to solve Eqs. (5), which leads to the following results:

$$a = a_0 (1 + \xi), \quad \phi = \phi_0 + \gamma, \tag{8}$$

where

$$\xi = \frac{\omega \mu_2 (\omega - \xi) \sin (\Omega t + \gamma)}{4 \sqrt{\frac{4 \xi^2 + \Omega^2}{\omega^2} + (\Omega^2 - \delta^2 - (\omega - \xi)^2)}}. \tag{9}$$

$$\gamma = \frac{\omega \mu_2}{4 \sqrt{4 \xi^2 + \Omega^2 + (\Omega^2 - \delta^2 - (\omega - \xi)^2)}} \sin (\Omega t + \gamma). \tag{10}$$

3. NONLINEAR ANALYSIS OF THE AMPLIFIER

In the second approximation the solution of Eq. (4) has the form

$$y = a \cos (\omega t + \phi) - \frac{a^2}{4} \sum_{n=1}^{\infty} \frac{1}{n} \cos \frac{2n}{12} \cos (\omega t + \phi). \tag{11}$$