GENERAL PROBLEMS OF METROLOGY AND MEASUREMENT TECHNIQUES

DESIGN OF AN ANALOG-TO-DIGITAL CONVERTER
WITH A CONSTANT RELATIVE ERROR

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The error of measurement for a digital instrument is dependent on the structure and circuit; it tends to vary over the scale and is generally represented by a three-term formula [1]. The maximum value \( \delta_{\text{max}} \) for the relative error falls within each measurement range. It is found [2] that a ratio of 10 may apply between the maximum value \( \delta_{\text{max}} \) observed at the start of a range to the minimum value occurring at the end of a range; this is not optimal from the information viewpoint, and therefore one needs to devise analog-to-digital converters with \( \delta_{\text{max}} = \text{const} = \delta_{\text{max}} \) for the entire range.

In general, \( \delta_{\text{max}} \) for an adc is a function of the probability that a given value of the measurement quantity \( L_x \) will occur [3], together with the accuracy specification for the measurements, so the specification of constancy in \( \delta_{\text{max}} \) can be realized only when each quantization step \( l_i \) will occur within certain limits for any current value [4]. One can adjust \( l_i \) after reducing the other sources of error as far as possible.

We consider some forms of digital instrument with \( 0 \leq \delta \leq \delta_{\text{max}} \) for \( L_{\text{min}} \leq L_x \leq L_{\text{max}} \).

A digital instrument with \( \delta_{\text{max}} = \text{const} \) for all \( L_{\text{min}} \leq L_x \leq L_{\text{max}} \) has been considered [5] for the case of absence of instrumental errors (apart from the quantization error) when the result of measurement \( L_{\text{res}} = L_x < L_{\text{max}} \) (Fig. 1) is taken as \( L_{\text{res}} = (L_{\text{res}} + L_{\text{ref}}) / 2 \); this has \( L_{\text{res}} = l_i \) for \( L_{\text{res}} > 1 \). Here \( L_{\text{res}} \) is the minimum value for a working measure and \( L_{\text{ref}} \) is the maximum adjacent value for the working measure, the difference of these being \( l_i \) (the subscripts \( s \) and \( e \) to \( L_i \) denote respectively the start and end of \( l_i \)). A real digital instrument always has instrumental errors which need to be taken into account in choosing the values of the measures.

Random Errors \( 0 \leq \delta \leq \delta_{\text{max}} \) Constant for All \( L_x \). For any \( L_x \) whose origin coincides with the origin of the set of values for the measure, the comparator determines the location of the end of \( L_x \) in the zone \( l_{\text{ai}} \) with a probability close to 1, and we assume for this that \( L_x = l_{\text{ai}} \) with an error \( 0 \leq \delta \leq \delta_{\text{max}} \) (the subscript \( a \) denotes an additive error) (Fig. 2a). The set of \( \{l_{\text{ai}}\} \) and \( \{l_{\text{ai}}\} \) for this instrument is defined as

\[
\frac{(l_{\text{ai}} + \Delta) \cdot 100}{L_{\text{ai}} - l_{\text{ai}} - \Delta} = \frac{(l_{\text{ai}} + \Delta) \cdot 100}{L_{\text{ai}} + l_{\text{ai}} + \Delta} = \delta.
\]

Here and subsequently the subscript \( \text{max} \) to \( \delta_{\text{max}} \) and \( \delta_{\text{max}} \) is omitted.

Then

\[ l_{\text{ai}} = l_{\text{ai}} + l_{\text{ai}} = l_{\text{ai}} \left( \frac{100 + \delta}{100 - \delta} \right)^{l-1} \]

and

\[ L_{\text{ai}} = \left[ L_{\text{ai}} - \frac{\Delta (100^2 - \delta^2)}{100 \delta} \right] \left( \frac{100 + \delta}{100 - \delta} \right)^{l-1} + \frac{\Delta (100^2 - \delta^2)}{100 \delta}, \]

and any value \( i \) of the working measure gives \( L_{\text{ai}} = l_{\text{ai}} - l_{\text{ai}} = L_{\text{ai}} = l_{\text{ai}} - \frac{100}{100 + \delta} + \Delta \).

For a set of working values for a measure in the case of multiplicative random errors (Fig. 2b) we have...
which gives

\[ \delta = \frac{(l_{si} + L_{is}) 100}{L_{Mi} - l_{si} - L_{is}} \text{ %} = \frac{(l_{se} + L_{ie}) 100}{L_{Mi} + l_{se} + L_{ie}} \text{ %}. \]  

(2)

Here and subsequently the subscript max to \( \delta_{\text{max}} \) is omitted, while subscript \( M \) denotes the multiplicative error, with \( L_{is}, L_{ie} \) the multiplicative errors at the points \( L_{is}, L_{ie} \); from (1) and (2) we have

\[ l_{mi} = l_{si} + l_{mei} = l_{ni} \left[ \frac{100 + \delta}{100 - \delta} \cdot \frac{100 - \delta}{100 + \delta} \right]^{\gamma - 1} , \]  

(3)

and for any value \( i \) of the working measure \( L_{Mi} = \frac{100}{100 - \delta} + L_{is}\).

Set of Values for a Working Measure with Additive Random Errors (0 \( \leq |\Delta| \leq |\Delta_{\text{max}}| \) = const) and Multiplicative Random Errors. Here (1) gives (Fig. 2c)

\[ \delta = \frac{(l_{msi} + L_{js} + \Delta) 100}{l_{m1} - m_{si} - L_{is} - \Delta} \text{ %} = \frac{(l_{mei} + L_{ie} + \Delta) 100}{l_{m1} + l_{mei} + L_{ie} + \Delta} \delta \]  

with the following equations that apply to all the cases (here and subsequently the lower case \( m \) denotes a mixed error:

\[ l_{msi} = \frac{l_{m1}[100 (\delta - \delta') - 8\delta'] - \Delta 100 (100 + \delta)}{(100 + \delta) (100 - \delta')} , \]  

(5)

\[ l_{mei} = \frac{l_{m1}[100 (\delta - \delta') + 8\delta'] - \Delta 100 (100 - \delta)}{(100 - \delta) (100 + \delta')} , \]  

(6)

\[ l_{mi} = l_{msi} + l_{mei} = l_{mi} \left[ \frac{100 + \delta}{100 - \delta} \cdot \frac{100 - \delta}{100 + \delta'} \right]^{\gamma - 1} . \]  

(7)