Behavior of the Energy Factor in the Bremsstrahlung Buildup Behind a Plane Layer of a Homogeneous Material

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It is shown that the energy factor in the bremsstrahlung buildup increases more slowly than linearly with the layer thickness, tending asymptotically toward the buildup factor for a monoenergetic source with the energy at which the attenuation factor reaches its minimum value in the concentrated part of the spectrum.

1. We denote by $B(E, H)$ the energy factor of the bremsstrahlung buildup behind a plane layer of a homogeneous material with an attenuation factor $\mu$ and a thickness $H$ for a plane, monoenergetic source of energy $E$. We denote the spectral distribution of the source intensity by $J_0(E)$. Then the energy factor for the buildup for a source with a continuous $J_0(E)$ spectrum can be written as

$$B(H, E_m) = \frac{\int_0^{E_m} B(E, H) J_0(E) \exp\left[-\mu(E)H\right] dE}{\int_0^{E_m} J_0(E) \exp\left[-\mu(E)H\right] dE}.$$ (1)

It follows from photon transport theory [1] that the energy factor $B(E, H)$ for a monoenergetic plane source is a rather smooth function of $H$ which increases approximately linearly without bound, i.e., it may be written as

$$B(E, H) = 1 + b(E, H) \approx 1 + a(E)H^{1/\varepsilon}, \quad \varepsilon \ll 1.$$ (2)

For substances which are not too heavy, function $a(E)$ in (2) is a monotonically decreasing function [1].

In this paper we will prove the following assertion: the (energy) factor of the buildup due to a plane source with a continuous spectrum characterized by a Schiff distribution (a monotonically nonincreasing spectrum for $E_m < 30$ MeV) increases more slowly than linearly as a function of $H$, tending asymptotically at large $H$ toward $B(E_0, H)$, $E_0 \in [0, E_m]$, where $E_0$ is the energy at which attenuation factor $\mu(E_0)$ reaches its minimum value; i.e., we have

$$\lim_{H \to \infty} B(E_m, H) = \lim_{H \to \infty} B(E_0, H).$$ (3)

This effect is important only for materials having moderate atomic numbers and for $E_m \approx E_0$. Figure 1 shows the dependence of the energy factor on the thickness of an iron layer for a Schiff spectrum with $E_m = 5$ and 10 MeV. The curvature of the $B(E_m, H)$ dependence can be explained in the following manner: the weight function

$$f_H(E) = \frac{\int_0^{E_m} J_0(E) \exp\left[-\mu(E)H\right] dE}{\int_0^{E_m} J_0(E) \exp\left[-\mu(E)H\right] dE}$$ (4)

is normalized to unity and gives, for fixed $H$, the contribution to $B$ of the various energetic components.

Fig. 1. Buildup factors corresponding to a plane, perpendicular source behind an iron layer: 1) for a Schiff spectrum with $E_m = 5$ MeV; 2) for a monoenergetic source with $E = 5$ MeV (interpolation from [1]); 3) for a Schiff spectrum with $E_m = 10$ MeV.

Fig. 2. The function $\Phi_H(E)$ behind an aluminum layer for the case in which the layer is irradiated with bremsstrahlung with $E_m = 5$ MeV (direct spectrum): 1) $H = 6$ cm; 2) 140 cm.

As $H$ increases, the maximum contribution moves toward spectral components having larger energies (Fig. 2). However, $B(E, H)$ is a decreasing function of $E$; i.e., for small $H$, the greatest weight is assigned to the low-energy components of the source spectrum, and at large $H$ the greatest weight is assigned to spectral components for which attenuation factor $\mu(E)$ is minimal. Equation (3) is proved rigorously in the Appendix.

**APPENDIX**

The discussion becomes simpler when the problem is formulated analytically.

**THEOREM.** Let function $B(E_m, H)$ be defined by Eq. (1), where $B(E, H)$ is a nonnegative linear function of $H$ and a monotonically decreasing function of $E$, and let $\mu(E)$ be a nonnegative, monotonically decreasing function. Then $B(E_m, H)$ increases more slowly than linearly as a function of $H$, and Eq. (3) holds. The proof follows from the auxiliary discussion which follows.

**LEMMA 1.** Let $F(E)$ be nonnegative, continuous, and nonvanishing on the interval $[0, E_m]$, and let the numbers $\alpha$ and $A$ satisfy the conditions

$$\alpha < 0, \quad A > 0.$$  \hspace{1cm} (1.1)

Then the family of functions

$$\Phi_H(E) = -\frac{F(E) \exp \left[ -AE^\alpha H \right]}{\int_0^{E_m} F(E) \exp \left[ -AE^\alpha H \right] dE}$$  \hspace{1cm} (1.2)

for which $H$ is a parameter forms a directionality which converges as $H \to \infty$ to a Dirac $\delta$-function $\delta(E - E_m)$.

**Proof.** From the conditions of the lemma it follows that $\Phi_H(E)$ has the following properties for all finite $H$: a) $\Phi_H(E)$ is continuous on $[0, E_m]$; b) $\Phi_H(E) > 0$; c) $\Phi_H = \int_0^{E_m} \Phi_H(E) dE = 1$. The continuous linear functional $\Phi$ on $(0, E_m]$ is continued by virtue of its continuity with conserved norm $\| \Phi_H \| = 1$ into the limiting function $\lim_{H \to \infty} \Phi_H(E)$, where

$$\lim_{H \to \infty} \Phi_H = \int_0^{E_m} \lim_{H \to \infty} \Phi_H(E) dE = 1.$$  \hspace{1cm} (1.3)