THE RADIATION FROM A CHARGE MOVING IN A PLANE WAVE FIELD AND A MAGNETIC FIELD

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The methods of classical electrodynamics are used to consider the problem of the radiation from an electron moving in the field of a monochromatic circularly polarized wave and a uniform magnetic field directed parallel to the direction of propagation of the wave. Expressions for the total radiation intensity and for the angular spectral distribution of the radiation are derived. Certain particular cases are studied.

§1. THE MOTION OF A CHARGE IN A PLANE WAVE AND A MAGNETIC FIELD

The classical problem of the motion of a negative charge in a plane-wave field with a steady uniform magnetic field directed parallel to the direction of propagation of the wave was solved in [1]. Let us consider a monochromatic circularly polarized wave, propagating along the unit vector \( \mathbf{n} \). Choosing the \( x \)-axis to lie along the vector \( \mathbf{n} \), we can expand the electric vector of the wave in a two-dimensional orthogonal basis \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \):

\[
E = E_0 (\mathbf{e}_1 \cos \omega t + \mathbf{e}_2 \sin \omega t), \quad \omega = \frac{t - \frac{z}{c}}{c},
\]

\[
(e_1, e_2) = (e_1 e_1) = (e_2 e_2) = 0,
\]

where \( g = \pm 1 \) describes the right-handed (\( g = 1 \)) and left-handed (\( g = -1 \)) polarization of the wave. It follows from (1) that the motion of an electron in such a field and in a uniform magnetic field \( \mathbf{H} (H = Hn) \) is described by the following parametric expressions:

\[
x = \frac{c}{\omega_0} \beta \cos \omega_0 \xi + \frac{1}{\omega_0 - \frac{eH}{m c \omega}} \cos \omega \xi,
\]

\[
y = \frac{c}{\omega_0} \beta \sin \omega_0 \xi + \frac{1}{\omega_0 - \frac{eH}{m c \omega}} \sin \omega \xi,
\]

\[
z = \frac{c}{\omega_0} \frac{1}{1 - \frac{eH}{m c \omega}} \sin (\omega - \omega_0) \xi,
\]

\[
t = \xi + \frac{z}{c}.
\]

In this equation \( \beta \) is an integration constant, related to the azimuthal velocity \( \omega_0 = c \beta \) of the electron in the absence of the wave, and the following notation is used:

\[
\gamma = \frac{E_0}{m c \omega}, \quad \omega_0 = \frac{eH}{m c \omega},
\]

\[
\gamma^2 = \left( \frac{c - ep_\mathbf{e}_z}{m c^2} \right)^2 = \frac{1 + \frac{e^2 p^2}{m^2 c^4}}{1 - \beta^2}.
\]

\[\text{in which } \epsilon \text{ is the total energy, and } p_z \text{ is the momentum of the charge along } z. \text{ The value of } \alpha \text{ is the integral of the motion.}\]

§2. THE TOTAL RADIATION INTENSITY OF THE CHARGE

The total radiation intensity of the charge can be calculated from formula (73.7) on page 253 of [2]. In our case the radiation intensity has the form

\[
\mathcal{W} = \frac{2e_1}{3m^2 c^3} \times \left[ \left( \frac{E_0}{\omega_0} \right)^2 + \frac{1 + \frac{e^2 p^2}{m^2 c^4}}{1 - \beta^2} \right] \cdot \delta(\omega - \omega_0).
\]

Evaluating the integral, we obtain

\[
\mathcal{W} = \frac{2e_1}{3m^2 c^3} \cdot \frac{1}{\omega_0^2} \times \left( \frac{E_0}{\omega_0} \right)^2 + \frac{1 + \frac{e^2 p^2}{m^2 c^4}}{1 - \beta^2} \right) + H^2 \beta^2.
\]

In the particular case of \( E_0 = 0 \), we obtain the well-known result for a magnetic field (see [3]). If \( H = 0 \), we obtain the total radiation intensity in the plane wave [4]. Notice that when \( \omega = \omega_0 \) there is a distinct resonance of the radiation intensity (\( \mathcal{W} \sim \infty \)). In this case the electron moves in an expanding spiral (see [1]).

Subsequently, having in mind electrons moving, for example, in accelerators (\( \omega_0 = 10^8 \text{ sec}^{-1} \)) and excited by light (\( \omega = 10^{15} \text{ sec}^{-1} \)), we shall consider the regions remote from resonance.

§3. THE ANGULAR SPECTRAL DISTRIBUTION OF THE RADIATION INTENSITY

To obtain formulas for the angular spectral distribution of the polarized radiation intensity, we follow the scheme proposed in [3] for synchrotron radiation. We write the vector potential of the radiation field in the form

\[
A = e_\mathbf{e}_0 \times \sum_{n,l} [e_n A_n (n, l) + e_0 A_0 (n, l)] e^{-i(\omega_0 t + \mathbf{e}_0 \cdot \mathbf{r})},
\]

where \( e_\mathbf{e}_0 \) are unit vectors in a spherical coordinate system. The polarized radiation intensity is then determined by the expression [5]

\[
d\mathcal{W} = \frac{e_1^2}{4\pi c} \sum_{n,l} l_2 A_n - l_3 A_0 \omega^2 d\Omega,
\]
where $I_x$ and $I_y$ represent different radiation polarizations (see [5]).

Calculating the radiation field potentials, we obtain

$$A_\nu (n, l) = \beta \frac{\partial F_{1,n}}{\partial \alpha} + \frac{\gamma}{\alpha} g \frac{\partial F_{1,n}}{\partial \beta},$$

$$A_\theta (n, l) = \text{ctg} \Theta F_{1,n} + \frac{\omega \left(1 - g \frac{\omega_0}{\omega}\right)}{\omega \sin \Theta} G_{1,n},$$

in which the functions $F_{1,n}$ and $G_{1,n}$ are

$$F_{1,n} = \sum_{s = -\infty}^{\infty} (-g)^s J_s (c) J_{l-s} (b) J_{s+\nu} (a),$$

$$G_{1,n} = \sum_{s = -\infty}^{\infty} (-g)^s J_s (c) J_{l-s} (b) J_{s+\nu} (a),$$

and the arguments $a$, $b$, $c$ are given by

$$a = \frac{\omega'}{\omega_0} \sin \Theta, \quad b = \frac{\gamma}{\alpha} \frac{\omega'}{\omega} \sin \Theta,$$

$$c = \frac{\gamma}{\alpha} \frac{\omega'}{\omega} \frac{1 - \cos \Theta}{1 - g \frac{\omega_0}{\omega}}.$$

The radiation frequency $\omega'$ is a function of $\omega$ and $\omega_0$:

$$\omega' = n \omega_0 + \omega_0, \quad n, l = 0, \pm 1, \pm 2, \ldots$$

Thus, formulas (5)–(8) completely determine the angular spectral distribution of the polarized radiation intensity.

The greatest practical interest arises in the case of a weak electromagnetic wave ($\gamma \ll 1$). In actual experiments with laser beams, $\gamma \approx 10^{-3}$). Expanding (6) and (7) in terms of $\gamma$, we obtain the square of the vector potential components:

$$A_1^2 = \beta^2 J_1^2 (n \beta \sin \Theta) b_{1,0} +$$

$$+ \frac{\gamma^2}{2 \alpha} \left(1 - g \frac{\omega_0}{\omega}\right)^2 \left(\Phi_\nu b_{1,0} + \psi_\nu b_{1,1}\right),$$

$$A_2^2 = \text{ctg}^2 \Theta J_2^2 (n \beta \sin \Theta) b_{2,0} +$$

$$+ \frac{\gamma^2}{2 \alpha^2} \left(1 - g \frac{\omega_0}{\omega}\right)^2 \left(\Phi_\nu b_{2,0} + \psi_\nu b_{2,1}\right),$$

$$\Phi_\nu = -\beta a \frac{\omega_0}{\omega} (1 - \cos \Theta) J_0 (n \beta \sin \Theta) \times$$

$$\times \left\{ \beta a \frac{\omega_0}{\omega} \left[1 + \cos \Theta + \frac{\beta^2 (1 - \cos \Theta)}{1 - g \frac{\omega_0}{\omega}} - \frac{2}{1 - g \frac{\omega_0}{\omega}} \right] \times$$

$$\times J'_0 (n \beta \sin \Theta) + 2 \text{ctg} \Theta J_0 (n \beta \sin \Theta) \right\},$$

$$\Phi_\theta = \alpha \text{ctg}^2 \Theta (1 - \cos \Theta) J_1 (n \beta \sin \Theta) \times$$

$$\left\{ \beta a \frac{\omega_0}{\omega} \left[1 + \cos \Theta + \frac{\beta^2 (1 - \cos \Theta)}{1 - g \frac{\omega_0}{\omega}} - \frac{2}{1 - g \frac{\omega_0}{\omega}} \right] \times$$

$$\times J'_1 (n \beta \sin \Theta) + 2 \text{ctg} \Theta J_1 (n \beta \sin \Theta) \right\}.$$