CALCULATION OF EFFECTIVE THERMAL RADIATION ABSORPTION COEFFICIENT OF A CAVITY WITH DIFFUSELY REFLECTING WALLS

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The system of integral equations of radiation heat exchange in a closed cavity is solved numerically.

One of the basic requirements of a calorimeter for radiant heat fluxes is the total absorption of all radiation incident on its entrance opening, independently of the spectral composition and direction of the radiation. The most effective method of increasing the absorption of radiation is the use of cavities of different configurations to collect the radiation. The geometry of a cavity can be changed so as to make its radiation characteristics approach those of a black body as closely as possible. The actual characteristics of the cavity can be determined either experimentally or theoretically, but the experimental arrangements for determining the absorptance of a cavity are so complex that only the theoretical solution of this problem is practical.

The effective thermal radiation absorption coefficient of a cavity of any configuration is defined as the ratio

\[ \varepsilon_{\text{eff}} = 1 - \frac{Q_{\text{ref}}}{Q_{\text{in}}} \]  

(1)

where

\[ Q_{\text{ref}} = \sum_{i=1}^{N} \int_{A_0} \left( \varphi_i(r_i) - f_i(r_i) \right) K_{ij} dA_j dA_0 \]  

(2)

is the reflected heat, and

\[ Q_{\text{in}} = \int \varphi_0(r_0) \, dA_0 \]  

(3)

is the incident heat. Here \( \varphi_i(r_i) \) is an unknown function characterizing the flux density of effective radiation from the \( i \)-th zone of the cavity surface (the subscript 0 refers to the opening); \( f_i(r_i) \) is a known function which characterizes the self-radiation of the cavity surface.

In order to find the unknown function \( \varphi_i(r_i) \), and consequently to determine the radiation characteristics of the cavity, it is necessary to solve the radiation heat-exchange problem in a closed cavity. By using the generalized zonal method this problem is reduced to the solution of a system of integral equations of the form [1]

\[ \varphi_i(r_i) = g_i(r_i) + \lambda_i \sum_{j=1}^{N} \varphi_j(r_j) K_{ij} dA_j \]  \( i = 1, 2, \ldots, N \).

(4)
where \( \lambda_i = 1 - \varepsilon_i \) is the reflection coefficient of the \( i \)-th zone of the cavity surface; \( g_i(r_i) \), distribution of thermal flux density or temperature over the \( i \)-th zone (specified in the boundary conditions); \( K_{ij} \), kernel of the integral equation, and is related to the elementary diffuse angular coefficient by the expression \( d F dA_i dA_j = K_{ij} dA_j \). For the geometry of the system shown in Fig. 1 the quantity \( d F dA_i dA_j \) is given by the expression

\[
d F dA_i dA_j = \frac{(n_i r_{12})(n_i r_{13})}{\pi r_i^4} \ dA_i.
\]

As shown in [2], solving Eq. (4) is equivalent to finding the extremum of the functional

\[
\mathcal{I} = \sum_{i=1}^{N} \int \phi_i^2 K_{i}dA_i dA_j + 2 \sum_{i=1}^{N} \sum_{j=1}^{i-1} \int \phi_i \phi_j K_{ij} dA_i dA_j \quad - \sum_{i=1}^{N} \int \phi_i^2 dA_j + 2 \sum_{i=1}^{N} \int g_i \phi_i dA_j.
\]

If the functions \( \phi_1, \ldots, \phi_N \) are determined so as to make the functional (5) an extremum, they are also the solution of the system of integral equations (4). Since it is difficult to find the exact solutions for the functions \( \phi_1(r_1), \ldots, \phi_N(r_N) \), we use the Ritz approximation method [3] in which each of the functions \( \phi_i(r_i) \) is represented as a linear combination of \( M \) appropriately chosen functions \( \psi_{im}(r_i) \):

\[
\phi_i(r_i) = \sum_{m=1}^{M} c_{im} \psi_{im}(r_i) \quad (i = 1, 2, \ldots, N),
\]

where the constants \( c_{im} \) are found from the condition

\[
\frac{\partial \mathcal{I}}{\partial c_{im}} = 0 \quad (i = 1, 2, \ldots, N, \ m = 1, 2, \ldots, M).
\]

This system contains \( MN \) algebraic equations with \( MN \) unknown coefficients. The accuracy of the solution obtained can be increased by increasing the number of terms in expansion (6).

The method described is used to calculate the effective absorption coefficient of the cylindrically symmetric composite cavity shown in Fig. 2. Equations (4) are solved subject to the following assumptions and boundary condition:

1) the quantities \( \lambda_1 = \lambda_2 = \lambda_3 = \lambda \) do not depend on the wavelength or direction of the incident radiation;
2) radiation is reflected and emitted diffusely by the cavity walls;
3) \( f_i(r_i) = 0 \), the self-radiation of the cavity walls is eliminated from the solution;
4) the opening is replaced by an ideal black surface with a uniformly distributed effective radiation flux density;
5) the basis functions in expansion (6) are chosen in the form

\[
\psi_{im}(r_i) = l_{i}^{m-1},
\]

where \( l_1 = r_1, l_2 = r_2, l_3 = r_3 \). The subscripts correspond to the numbers of the surface shown in Fig. 2.

Using the assumptions and boundary conditions, we obtain from (5)-(7)

\[
AX = B,
\]

where \( X \) is a column vector of the unknown coefficients

\[
x_k = c_{jmn}, \ k = (j - 1) \times M + n \quad (j = 1, 2, 3; \ n = 1, 2, \ldots, M);
\]

\( A \) is the matrix of the coefficients with elements

\[
a_{jk} = \begin{cases} 2 \int \int \left( l_{1}^{m-1} l_{2}^{n-1} K_{1j} dA_1 dA_j \quad \text{for} \ i \neq j, \\ \frac{2}{\varepsilon_1} \int \int \left( l_{1}^{m+n-2} dA_1 dA_1 \quad \text{for} \ i = j; \end{cases}
\]

where \( \varepsilon_1 \) is the emissivity of the cavity.