MAGNETOOPTICAL EFFECTS IN FERROMAGNETIC METALS AT LOW TEMPERATURES

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Zeroth-order and first-order Boltzmann kinetic equations are found for the frequency-dependent transverse-conductivity tensor for inelastic scattering of electrons, with an account of thermal oscillations of the scattering centers. At low temperatures this tensor, which is linear in the spin-orbit interaction, depends exponentially on the temperature. A relation is found between the magnetooptical effects in ferromagnetic metals and the topology of the Fermi surfaces; from this relation, qualitative conclusions can be drawn regarding the effect of this topology on the magnetooptical effects and regarding the sign of the anomalous magnetooptical parameter (which depends on whether hole or electronic conductivity predominates).

There have been several theoretical studies of magnetooptical effects in ferromagnetic metals [1-5]; Boltzmann equations have been obtained in the lowest approximation (neglecting the spin-orbit interaction), and higher-order equations in the scattering potential, linear in the spin-orbit interaction, have been obtained in efforts to find the frequency-dependent conductivity tensor for elastic scattering of current carriers by scattering centers in the ordinary one-electron approximation and for the case of Bloch electrons.

For both practical and theoretical calculations, it has been assumed that a relaxation time exists which is independent of the frequency $\omega$ of the external force. However, the assumption that this time is independent of the frequency is questionable: the relaxation time is in fact a function of the frequency of the external force, and this dependence of $\tau$ on $\omega$ is displayed in the reflection properties of metals.

In efforts to find the complex frequency tensor of the conductivity, the randomly distributed impurity centers are usually treated as static defects at rest. It is assumed that the transition probabilities per unit time $\omega^{(0)}_{ll'}$ and $\omega^{(1)}_{ll'}$ of the lowest and highest orders, etc., are symmetric with respect to $l$ and $l'$ and are consistent with the assertion that the relaxation time is independent of the energy spectrum of impurity atoms (zero-point oscillations). This procedure leads to a temperature-independent residual resistivity $\rho_{res}$, which is also dubious.

In real crystals, the impurities vibrate, and the nature of their vibrations differs significantly from that of the matrix atoms. Because of the zero-point vibrations, $\rho_{res}$ depends on the vibration spectrum and may be nonvanishing even when the impurity atoms have the same scattering amplitude as the matrix atoms [6]. Then conduction electrons are scattered inelastically by impurities, and this scattering is reflected in the transition probability $(\omega_{ll'}^{(0)}$ $\neq$ $\omega_{ll'}^{(1)})$ and thus in $\tau$. It seems that the inelastic nature of the scattering of conduction electrons by impurity centers should be taken into account in finding the conductivity frequency tensor for low temperatures.

Below we will briefly discuss magnetooptical effects in ferromagnetic metals during inelastic scattering of electrons by impurities. To find the conductivity $\tilde{\sigma}_{\alpha\alpha}(\omega)$ and the complex transverse conductivity tensor $\tilde{\sigma}_{\alpha\alpha}(\omega)$, we find the corresponding kinetic equations and solve them, taking account of the thermal-vibration spectrum of the impurities. The dependence of the transverse conductivity tensor $\tilde{\sigma}_{\beta\alpha}(\omega)$ on $\omega$ and $\tau$ at low temperatures is demonstrated. A relation is found between the anomalous magnetooptical effect and the structure of the Fermi surface; from this relation, qualitative conclusions can be drawn regarding the sign of the magnetooptical parameter.
The kinetic equations derived in [7] by the density-matrix method, and generalized in [8] and by the authors, for the inelastic scattering of electrons by impurity centers are

\[
\begin{align*}
\rho \frac{\partial \rho}{\partial t} &= i E_\alpha \frac{\partial \rho}{\partial \varepsilon} + 2\pi \kappa \sum_N \rho_N \sum_{N'} \left[ \left| \phi_{NN'} \right|^2 \delta (\omega_{NN'} - \omega) \right. \\
&\left. - \omega \phi (1 - f_i^{-2}) \right] f_i^{-2} (1 - f_i^{-2}) (1 - f_i^{-2}) \sum_N \rho_N \sum_{N'} \left| \phi_{NN'} \right|^2 \delta (\omega_{NN'} - \omega) f_i^{-2} (1 - f_i^{-2}) \right],
\end{align*}
\]

where

\[
\begin{align*}
\omega_{NN'} &= \varepsilon_N - \varepsilon_{N'}, \\
\omega &= \varepsilon_N - \varepsilon_{NN'}, \\
\omega &\Rightarrow \omega_{NN'}, \\
\delta (\omega_{NN'} - \omega) &= \delta (\omega_{NN'} - \omega), \\
DN &\Rightarrow DN', \\
D &\Rightarrow DN'.
\end{align*}
\]

\( \rho_N \) is the impurity density matrix, \( \varepsilon_N \) and \( \varepsilon_{NN'} \) are the eigenvalues of the operators corresponding to the impurity and electron kinetic energies, and \( \phi \) is the impurity-electron interaction potential. The summation over \( N \) in Eqs. (1) and (2) is carried out over all the impurity centers in the system.

We see that Eqs. (1) and (2) differ from the corresponding equations in [4, 5], where elastic scattering was treated and where the impurity energy spectrum was neglected. Equation (1) is a generalization of the ordinary Boltzmann equation which can be used to find the complex frequency dependence of the conductivity in the case of inelastic scattering and a Fermi-Dirac distribution, while Eq. (2) is the first correction to generalized kinetic equation (1).

Retaining in (2) only the terms linear in the spin-orbit interaction, and proceeding as in [5, 9, 10], we find the solutions of (1) and (2) to be

\[
\begin{align*}
f_i^{-2} &= -e E_\alpha \frac{\tau}{1 + i \omega \tau} \phi, \\
\phi (1 - f_i^{-2}) &= \frac{1}{\tau} \left( \frac{\partial f_i}{\partial \varepsilon} - \frac{\partial f_i}{\partial \varepsilon} \right) \exp \left[ - \frac{\omega_{NN'} + \omega}{kT} \right],
\end{align*}
\]

where \( \rho_i = \partial f_i / \partial \varepsilon_i \); \( \tau \) and \( \tau_0 \) are the relaxation times for conduction electrons for inelastic and elastic scattering by impurity centers, respectively; \( J_\alpha \) is the diagonal part of the matrix element of the coordinate linear in the spin–orbit interaction; \( n \) is the number of impurity centers per unit volume; and \( \varepsilon \) is the average impurity potential. It follows from (3) for the relaxation time for the case of inelastic scattering that we have \( \tau_1 \sim \exp \left( -\frac{\omega_{NN'} + \omega}{kT} \right) \) at low temperatures, while \( \tau_0 \sim \rho_{res} \) does not depend on the temperature. The \( T \) dependence of \( \tau_1 \) is particularly important during scattering by impurities under the influence of a variable, i.e., frequency-dependent, external force. As \( T \rightarrow 0 \), \( \tau_1 \rightarrow \tau_0 \) governs the residual resistivity as a function of the frequency of the external force, while for \( T \ll 1 \), \( \tau_1 \) becomes temperature-dependent. As the temperature increases, the amplitude for purely elastic scattering decreases; this behavior is particularly evident when the external force is not constant.

It follows from a comparison of (5) with the corresponding equation for \( \tau_1 \) in the case of a very slowly varying electric field \( E_\alpha (t) = E_0 e^{i\omega t} \) (where \( \omega > 0 \) is a small parameter) that while with \( E_\alpha \sim E_0 e^{i\omega t} \) an account of inelastic scattering, i.e., the impurity-atom spectrum, plays an important role at very low temperatures, account of the impurity spectrum with \( E_\alpha \sim E_0 e^{i(\omega + \delta) t} \) plays an important role over a broad temperature range.