A model at any of these levels should be open, i.e., the mathematical formalization should allow one to incorporate additional units that either describe different processes or else refine the couplings between elements. Also, the mathematical techniques and the logic of the couplings should provide, if necessary, for combining models at the different levels in application to any particular system.

**NOTATION**

T, temperature; r, spatial coordinate; n, normal; t, time; λ, thermal conductivity; ρ, density; c, specific heat; c_m, mass heat capacity of a lumped element; k_{ij}, conductivity between elements i and j of model; q_r, source function; q_{ext}, heat flux from the surroundings to the element; q_{int}, heat flux characterizing thermal interaction of elements; D_j, space region corresponding to element j of model; Γ_j, boundary of D_j; Γ_{ext} and Γ_{int}, sets of external and internal boundaries; N and N_a, numbers of distributed and lumped elements; subscripts: i, j, model element; α, lumped element.

**CONCEPTUAL ALGORITHMS FOR ANALYSIS OF EXPERIMENTAL DATA**

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A study is made of process simulation in inverse situations. Some problems arising in this approach are discussed, and a study is made of the choice of solution form, as well as of solution technique.

An important aspect of computer assistance in thermophysical research is to design algorithms for interpreting results; inverse treatments are often involved [1, 2]. In the treatment of experimental data, one often has to determine the causes of an observed effect, and inverse treatments provide a basis for analysis, which involves concepts in the interpretation of data. In practice, one often has the necessary information on the object in the form of models for conservation laws. Incorporation of a model into an analysis algorithm provides for more profound study of the structure and relevant factors. The corresponding algorithms may therefore be called conceptual, since the analysis is performed by inverse simulation. Simulation involves transfer from a general functional description to some particular description, which distinguishes this approach from other ways of solving inverse problems. In other words, conceptual algorithms presuppose the solution of more general problems, in which the formalization applies to the model and not to the initial data. The result is a model for an experiment that can provide characteristics of the process that are not accessible to direct observation, including the dynamic behavior of the object and so on.

We now consider ways of designing conceptual algorithms. We assume that we have chosen a model

\[ L_0 u = f, \]  

and some observations are given

\[ u_0 = \bar{u} + \varepsilon \]

with a known value for the norm of the deviation from the true value:

\[ ||u_0 - \bar{u}|| \leq \delta. \]  

In the choice of the model of (1) it is assumed that there exists a solution \( u \in U \) that is unique in some metrical space \( U \) and that is continuously dependent on the initial data \( a \in H \) and \( f \in F \).

The model of (1) is the result of formalizing the process, so some of the parameters may be unknown or may differ from the actual object parameters. If appropriate a priori estimation is difficult, these parameters may be included in a vector \( a \). Then the conceptual processing amounts to simultaneous determination of the state function \( u \in U \) that satisfies the model throughout the relevant region of the independent variables together with the unknown parameters in metrical space \( H \), which is the space of vector \( a \). The latter requires additional information, where condition (2) is used.

In the construction of such a solution, it may happen that the result is not unique or that the solution is unstable. In the first case it is then necessary to establish a one-to-one correspondence between the desired quantities and the given
sample, which enables one to recover the causes from the effect. In the second case, it becomes necessary to determine why the solution is not continuously dependent on the initial data. In such a case one should search for a method that can be applied to a broad class of inverse problems with varying levels and distributions for the errors of measurement, and which also allows one to convert from a general functional representation to a particular form. These specifications are met by regularization as described below.

We assume that the conditions for one-to-one correspondence are met and reduce the problem to

$$S\mathbf{a} = \mathbf{b}, \tag{3}$$

where $S$ is an operator that performs a mapping from $\mathbb{H}$ to $\mathbb{U}$ and which is specified inexplicitly by the form of model (1).

The errors of observation $e$ mean that the solution to (3) is unstable. Then the general theory of regularization [3] indicates that the solution must be sought in a space with a norm not weaker than the norm of the observation space. In practice, observations constitute a discrete set $\mathbf{b} = \{b_{ij}\}_{i=1}^{n}$ at $n$ points of measurement for each of the $m$ points of observation. Such samples may correspond to a space with a Euclidean norm. Therefore a stable algorithm for solving (3) for real number spaces, when $\mathbb{H} = \mathbb{E}_p$, may be based on the following regularization:

$$\min_{\mathbf{a} \in \mathbb{E}_p} \max_{k \in \{1, \ldots, m\}} \sum_{i=1}^{n} (b_{ij} - a_{ik})^2 \leq \delta_i^2, \quad i = 1, \ldots, m, \tag{4}$$

where $u$ is a function that satisfies model (1) with given $a$, $i$ is the number of the observation point, and $j$ is the instant of measurement.

Regularization can be performed as follows if another form is chosen for the error of measurement, e.g., the maximum deviation from the true value:

$$\min_{\mathbf{a} \in \mathbb{E}_p} \max_{k \in \{1, \ldots, m\}} \max_{i=1}^{n} \|u_{ij} - a_{ik}\| \leq \delta_i, \quad i = 1, \ldots, m. \tag{5}$$

Therefore, conceptual analysis of models in which the unknown parameters are independent of the state function can be performed via the minimax problems of (4) or (5), in which one defines an element in the space of real numbers in conformity with the error of the input data. The criterion in (4) and (5) is the stabilizing functional

$$\Omega[a] = \max_k |a_k|,$$

and the $\delta_i$ denote the estimate of the error of measurement for element $i$ in the observation set. Any change in the error estimator for a given set is reflected in the form of the coupling conditions, while the general mode of solution remains unchanged:

$$\min_{\mathbf{a} \in \mathbb{H}} \Omega[a], \tag{6}$$

$$\|u_{ij} - a_{ik}\| \leq \delta_i, \quad i = 1, \ldots, m,$$

where $u_i$ is the value of the solution to (1) at the point of observation $i$ and $\| \cdot \|$ is the norm corresponding to the form of the measurement-error estimator.

Other regularization schemes have been proposed [4, 5], but they differ in that the formulation of (6) is based on a particular use of the elements from the observation set $\mathbf{b}$, and a match is made to the error of the input data at each observation point separately, not via the overall dispersion. This reflects the form of observation common in practice. On the other hand, the formalization of (6) demonstrates a general approach to determination of the coefficients in the equation and to the conditions at the boundaries of the object.

This approach may be demonstrated in the solution of the following model problem. We assume that a physical process is described by the boundary-value problem

$$\frac{\partial u}{\partial t} = \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + x + t - x^3, \quad x \in (0, 1), \quad t \in (0, T), \tag{7}$$