Both analog and digital communication channels (ACC and DCC) are used in up-to-date Information Measuring Systems (IMS). Below we examine pulse-code modulated (PCM) IM systems which utilize channels of both types. The measured quantity \( x(t) \) (random stationary process) is transmitted along a given ACC (Fig. 1 shows a single measuring channel of the system) through the multiplexor switch MS to the analog–digital transducer ADT. The signal \( y(t_i) \) is converted into the number \( y_i \) (code combination), which is transmitted along the DCC through the multiplex-decoding switch MDS to the restoring device (RD). The latter builds up a continuous process \( z(t) \) from the received numbers \( \hat{y}_i \).

The ACC normally comprises a wire (cable) communication line and an autonomous (normalizing) transducer. The DCC and RD can consist of different devices, for instance, of a digital recorder.

Considering the RD to be a stepped interpolator, let us represent the error \( \Delta(t) = z(t) - x(t) \) of a system with a single ideal (desired) operator in the form

\[
\Delta(t) = \Delta_1(t) + \Delta_2(t) + \Delta_3(t),
\]

where \( \Delta_1(t) = k_s y(t_i) - x(t) \) is the linear-section error; \( \Delta_2(t) = k_s [y_i q - y(t_i)] \) is the ADT error; \( \Delta_3(t) = z(t) - y_i q k_s = q k_s (y_i - y_i) \) is the digital-section error; \( k_s \) is the scale factor (RD transfer factor); \( q \) is the quantization increment.

The above component representations can be used for evaluating the system error from experimental investigations of its component parts (sections) or theoretically from the components (units) characteristics.

From the given metrological characteristics of the primary transducer PT, ACC, and MS it is possible to find the pulse transfer function \( g(t) \) of the linear section and the interference signal \( \xi(t) \) at its output [1]. The ACC should then be considered as a linear stationary link with lumped parameters. Bearing in mind the weak correlation among \( \Delta_2(t), \Delta_3(t), \) and \( \Delta_1(t) \), as well as the proximity to zero of their mathematical expectations [2, 3], we find on an average over a quantization increment that

\[
m_{\Delta} = m_{\Delta} = (t_i b - 1) + k_s m_s,
\]

\[
\sigma_{\Delta}^2 = k_s^2 \sigma_{\xi}^2 - 2 k_s \int_0^T g(t) \rho_x (t + \tau) d \tau d \tau + \sigma_{\Delta_2}^2 + \sigma_{\Delta_3}^2,
\]

where \( m_{\Delta}, \sigma_{\Delta}^2 \), \( \sigma_{\xi}^2 \) and \( \sigma_{\xi}^2 \) are the mathematical expectations and dispersions of \( x(t) \) and \( \xi(t) \); \( \rho_x (\tau) \) is the autocorrelation function of \( x(t) \).

In finding the minimum of \( \sigma_{\Delta}^2 \) with respect to \( k_s \) we obtain

\[
\sigma_{\Delta_0}^2 = \min_{k_s} \sigma_{\Delta}^2 = \frac{\sigma_{\xi}^2}{\sigma_{\xi}^2 + \sigma_{\Delta_2}^2 + \sigma_{\Delta_3}^2} \times \left[ \frac{1}{T} \int_0^T g(t) \rho_x (t + \tau) d \tau d \tau \right]^2,
\]

and the optimum value of \( k_s \) is

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Let us note that expressions approaching (1) and (2) were obtained in [4]. If \( m_\alpha \neq 0 \) and \( m_\varepsilon \neq 0 \), then the process \( z(t) \) becomes a biased evaluation of \( x(t) \) for \( k_\infty = k_{\infty 0} \). Therefore, it is advisable to apply to the measurement results the correction \( m_\Delta \). The fact that \( k_{\infty 0} \) depends on the IMS systems' metrological characteristics indicates that, in a general case, a minimum system errors dispersion is not obtained by minimizing various component-error dispersions.

If \( x(t) \) is a low-frequency signal and the width of its spectrum is considerably smaller than the passband of the linear section, i.e., if

\[
\sigma_x^2 \int_0^\infty \rho_x(t_1-t_2) g(t_1) g(t_2) dt_1 dt_2 \ll b^2 \sigma_\xi^2 - 2b\sigma_\sigma^2 \int_0^\infty \rho_x(t) g(t) dt \ll \sigma_\Delta^2,
\]

then

\[
\sigma_y^2 = b^4 \sigma_x^2 + \sigma_\xi^2,
\]

\[
\frac{1}{T} \int_0^T \int_0^T g(t) \rho_x(t + \tau) dt d\tau = \frac{b}{T} \int_0^T \rho_x(t) dt,
\]

where \( b = \int_0^\infty g(t) dt \) is the linear-section constant-signal transfer factor and

\[
\sigma_{\Delta 0}^2 = \sigma_x^2 - \frac{b^4 \sigma_\xi^2}{b^4 \sigma_x^2 + \sigma_\xi^2 + \sigma_{\Delta 2}^2 + \sigma_{\Delta 3}^2} \times \left[ \frac{1}{T} \int_0^T \rho_x(t) dt \right]^2.
\]

By representing \( \rho_x(t) \) in the interval of \( 0 \leq \tau < T \) as \( \rho_x(t) = 1 - a_j t^j \) [1, 5], it becomes possible to reduce (3) to the form

\[
\epsilon_{\Delta 0}^2 = \frac{\sigma_{\Delta 0}^2}{\sigma_x^2} = \epsilon_\xi^2 + \epsilon_{\Delta 2}^2 + \epsilon_{\Delta 3}^2 + 2 \frac{a_j T^j}{j + 1},
\]

where

\[
\epsilon_\xi^2 = \sigma_\xi^2 / b^2 \sigma_x^2; \quad \epsilon_{\Delta 2}^2 = \sigma_{\Delta 2}^2 / b^2; \quad \epsilon_{\Delta 3}^2 = \sigma_{\Delta 3}^2 / b^2 \sigma_x^2.
\]

Moreover,

\[
b^2 \sigma_x^2 \gg \sigma_\xi^2 + \sigma_{\Delta 2}^2 + \sigma_{\Delta 3}^2, \quad a_j T^j \ll 1.
\]

If \( \epsilon_\xi^2 < q \), then we find from [6] that

\[
\epsilon_{\Delta 4}^2 = \frac{\mu^2}{12} 4^{-n} + \epsilon_{\Delta 4}^2,
\]

where \( \mu = L / \sigma_x \), \( L \) and \( n \) are the range and the number of ADT binary orders; \( \epsilon_{\Delta 4}^2 \) is the ADT internal-noise dispersion normalized with respect to \( b^2 \sigma_x^2 \) and determined according to [7].

The error \( \Delta_3(t) \) is due to pulse distortions in the IMS digital section (decoding errors). It is shown in [8] for binary codes which have independent distortions with the mean probability \( p \) that

\[
\epsilon_{\Delta 3}^2 = \frac{\mu^2}{3n} \sum_{k=0}^{m} \frac{m-1}{2^k} \frac{m+1}{2^k} p.
\]