A solution is presented for a transient two-dimensional problem of the hydrodynamics of a compressible non-Newtonian fluid connected with the propagation and damping of shock waves in a well.

The physical processes connected with well drilling have been studied in increasing detail in recent times, a fact related to the seriousness of the consequences of emergency situations at oil and gas extraction sites. Theoretical study of the hydrodynamics of wells is complicated by the need to solve problems for non-Newtonian fluids — which includes most drilling fluids. Here, most of the work that has been done has examined unidimensional and quasi-unidimensional hydrodynamic problems, with investigators neglecting or averaging two-dimensional and nonlinear effects [1, 2].

Examined below is a two-dimensional transient problem of the hydrodynamics of a compressible non-Newtonian fluid with allowance for several nonlinear phenomena which might exert a marked effect in the generation and propagation of shock waves in long channels. We chose for the form of the rheological equation a relation describing shear stress as an exponential function of shear rate, which is a good approximation for most drilling fluids.

Given this model, we may study a whole range of problems of dynamics connected with the opening up of beds with pressure anomalies, the closing of pipe connections, start-up of pumps, lowering and raising of drilling equipment, etc. Here, we examine the first of these problems and solve it by the method of fractional steps [3] on an R-1040 computer.

The unsteady motion of a non-Newtonian fluid is described by the following dynamic equation [4, 5]:

\[
\frac{\partial \gamma}{\partial t} + \rho \nabla \cdot \gamma = -\nabla p + \nabla \tau \cdot + \rho g. \tag{1}
\]

In the case of a compressible fluid, apart from the shear stresses, the viscous stress tensor should also account for linear strain [4], i.e.,
We assume that for the fluid in question there is assigned a nonlinear rheological equation of state which provides for agreement between \( \tau \) and the tension tensor \( (D) \). Here, \( \tau' = \tau \), since the liquid was assumed to have been incompressible \( (\text{div} \, v = 0) \) in deriving the rheological equation

\[
\tau' = g(D),
\]

where \( D = \frac{1}{2} (\gamma \nu + \nu \nu^T) \). The specific form of this relation for the so-called generalized non-Newtonian fluid \([4]\) is expressed in terms of the second principal invariant of the tensor

\[
\tau' = 2\eta(S) D,
\]

where \( S = 2 \text{tr}(D^2) \), here \( \text{tr} \) is the trace of the tensor.

The value of \( \eta \) is assumed constant for a Newtonian fluid, and Eq. \((1)\) converts to the Navier–Stokes equation. Substitution of Eq. \((2)\) into \((1)\) with allowance for \((3)\) leads to the generalized form of the Navier–Stokes equation \([4, 5]\):

\[
\frac{\rho}{\partial t} \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P + \rho g + \eta(S) \left\{ \mathbf{v} \nabla \mathbf{v} + \frac{1}{3} \nabla \cdot \mathbf{v} \right\} + 2 D \nabla \eta(S) \frac{2}{3} \nabla \cdot \mathbf{v}.
\]

Equation \((4)\) will be integrated with a power law of change in viscosity \([4]\), i.e., where

\[
\eta(S) = kS^{(n-1)/2}.
\]

We should note that drilling fluids are described well by relations of the type \((5)\) for \( n < 1 \). To close the system, together with Eqs. \((4)\) and \((5)\), we should examine the equation of continuity for a compressible fluid \([4]\):

\[
- \frac{\partial P}{\partial t} = \rho c^2 \nabla \cdot \mathbf{v},
\]

where \( c = \sqrt{k / \rho} \).

Let us now examine the following problem. Let a viscous fluid with a known rheology enter the bottom of a vertical tube (Fig. 1) of length \( L \) with an annular cross section of radii \( R_1 \) and \( R_2 \) (in particular, the internal radius can be \( R_1 = 0 \)). A steady flow is established in the tube with a profile which depends on the type of rheological law, the geometry of the channel, and the mass flow rate \([1]\). We will study the propagation of dynamic perturbations which arise with an abrupt change in the mass flow rate or the pressure at the ends of the channel in the case of the movement of a non-Newtonian fluid in the channel.

Let us choose a cylindrical system of coordinates, \((z, \varphi, r)\), with the \( z \) axis coinciding