EFFECT OF NONUNIFORM SENSITIVITY DISTRIBUTION ALONG A WIRE THERMOANEMOMETER PROBE ON TURBULENCE MEASUREMENT

A. I. Popov

The effect is determined of nonuniform distribution of temperatures along a wire probe on the averaged turbulent velocity pulsation along it.

When turbulent pulsations of velocity are measured a measurement error arises due to averaging over the length of the probe.

This error was previously analyzed by assuming uniform sensitivity distribution along the probe [1]. In actual fact, however, the sensitivity is distributed along the probe in a nonuniform manner due to temperature distribution along its length. The end of the probe becomes cooler due to massive holders and this results in a shortening of the effective length. The latter modifies the averaging effect over the probe on the thermoanemometer readings. The estimation of the effect of averaging of the velocity pulsation along the probe with nonuniform temperature distribution is the subject of this article.

It is assumed that by employing one of the well-known methods the heat drift has been eliminated or can be ignored.

In this case the temperature distribution along the probe thread is governed by the equation

$$\frac{d^2 \theta}{dx^2} - \frac{\theta}{\lambda_2} = -\frac{4Q}{\lambda_2 \pi d^2}, \quad (1)$$

where

$$a^2 = 4 \frac{Nu}{d^2} \frac{\lambda_1}{\lambda_2}; \quad Nu = -\frac{\alpha d}{\lambda_1}; \quad Q = \rho p; \quad Re = \frac{Vd}{v}.$$  

The Nusselt number can be found from the relation [2]

$$Nu = 0.42 Pr^{0.2} + 0.57 Pr^{0.33} Re^{0.5}. \quad (2)$$

The temperature and velocity are each represented by a sum of two components, the averaged and the pulsation one:

$$\theta = T + t, \quad V = U + u. \quad (3)$$

The averaged temperature T is constant in time but variable along the wire. The averaged velocity U is constant both in time and space. The variable components t and u are random functions of time and of the coordinate x. Moreover, it is assumed that u is a homogeneous function of the coordinate x.

One assumes, as it is customary in thermoanemometry, that the pulsating components (or more precisely, their mean-square values) are much smaller than the averaged quantities:

$$u \ll U, \quad t \ll T.$$  

Under this assumption the multiplier Re^{0.5} appearing in (1) can be made equal to


© 1975 Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for $15.00.
Re_0.5 = \sqrt{\frac{d}{v}} (U + u)^{0.5} \approx \sqrt{\frac{U d}{v} \left( 1 + 0.5 \frac{U}{u} \right)}.

(5)

Substituting (5) in (2) and then in (1), two equations are obtained for temperature distribution:

1) for averaged temperatures,

\[
\frac{d^2 T}{dx^2} - \alpha^2 T = \frac{4Q}{\kappa \pi d^2};
\]

(6)

2) for the pulsating temperature component,

\[
\frac{d^2 t}{dx^2} - \alpha^2 t = \varphi(x),
\]

(7)

where \( \varphi(x) = DT(x) u(x) \).

The coefficient \( \alpha \) appearing in (6) and (7) can be determined from (2) by using \( U \) only.

It has already been pointed out that the temperature at the ends of the wire is constant and equal to the temperature of the surrounding medium. Hence the boundary conditions for Eqs. (6) and (7) are given by

\[
T = t = 0 \quad \text{and} \quad x = \{ 0, l \}.
\]

(8)

The solution of Eq. (6) with the boundary conditions (8) is known and is of the form [2]

\[
T = D_1 \left[ 1 - \frac{\text{ch} a \left( x - \frac{l}{2} \right)}{\text{ch} \frac{al}{2}} \right].
\]

(9)

To find the distribution of the pulsating temperatures one has to solve Eq. (7) together with the boundary conditions (8). This solution is given by

\[
t = \frac{\text{sh} ax}{a} \left[ \sum_0^l \varphi(y) \left( \text{ch} ay - \text{sh} ay \right) + \int_0^x \varphi(y) \text{ch} ay \, dy \right]
\]

\[
- \frac{\text{ch} ax}{a} \int_0^x \varphi(y) \text{sh} ay \, dy.
\]

(10)

The tension between the ends of the wire is given as the product of the current and its resistance.

The relation between the resistance of the unit length of the wire and the temperature in the range of temperatures characteristic for thermoanemometry can be represented by a linear relation

\[
\rho = \rho_0 [1 + \beta (T + T_0)].
\]

(11)

The pulsating part of the resistance is obtained from (11), namely

\[
\tilde{\rho} = \rho_0 \beta t.
\]

(12)

The pulsating tension between the wire ends is given in terms of the pulsating temperature by the relation

\[
e = I \int_0^l \tilde{\rho} \, dx = I \beta \rho_0 \int_0^l t \, dx.
\]

(13)

Inserting (10) in (13) and by subsequent calculations one obtains the relation between the tension at the ends of the wire and the pulsation velocity,

\[
e_m = k \int_0^l \left[ \frac{\text{ch} a \left( x - \frac{l}{2} \right)}{\text{ch} \frac{al}{2}} - 1 \right]^2 u \, dx.
\]

(14)

If lack of uniformity in the velocity field along the wire can be ignored then (14) can be replaced by