DYNAMIC METHOD OF MEASURING THE THERMAL CONDUCTIVITY OF GASES AT HIGH TEMPERATURES

R. A. Mustafaev

The author proposes a method of measuring and a formula for calculating the thermal conductivity of gases during monotonic heating.

Thermophysical measurements are made basically on specimens of simple shapes (plate, cylinder, sphere) inside which a one-dimensional temperature field \( t(x, \tau) \) with a sufficiently small drop is artificially maintained.

The following nonlinear equation of heat conduction is valid for calculating the temperature field in such specimens:

\[
\frac{\partial^2 t}{\partial x^2} + \frac{1}{\lambda} \cdot \frac{dt}{d\tau} \left( \frac{\partial t}{\partial x} \right)^2 = \frac{1}{a} \cdot \frac{\partial t}{\partial \tau} .
\]  

(1)

Generally, the coefficients \( a(t), \lambda(t), c(t), \) and \( \gamma(t) \) are arbitrary functions of the temperature.

An analytic solution can be obtained only in individual special cases where the temperature-dependence of the thermophysical properties is subject to simplifying constraints and where either rigorous or approximate transformations may reduce the equation to a linear one.

The simplest case is

\[
a(t) = \text{const}; \quad \lambda(t) = \text{const}; \quad c(t) = \text{const}; \quad \gamma(t) = \text{const}.
\]  

(2)

With (2), Eq. (1) transforms into the ordinary linear equation of heat conduction

\[
\frac{\partial^2 t}{\partial x^2} = \frac{1}{a} \cdot \frac{\partial t}{\partial \tau},
\]  

(3)

which has been solved in [1] for various different boundary conditions.
Almost all existing methods of thermophysical measurements are based on regularities in the solution to Eq. (3). With any other assumptions concerning the functional relations $a(t)$, $\lambda(t)$, $c(t)$, and $\gamma(t)$ than those in (2), Eq. (2) will remain nonlinear and will require approximate methods of solution.

Thermophysical measurements usually involve a small temperature drop $\delta(r, \tau)$. This allows us to represent the parameters $a$, $\lambda$, $c$, and $\gamma$ on intervals commensurable with $\delta(r, \tau)$ as Taylor series expansions of the respective functions in powers of $\delta(r, \tau)$:

$$
\begin{align*}
    a &= a_0 + k_a \delta + n_a \delta^2 + \cdots; \\
    \lambda &= \lambda_0 + k_\lambda \delta + n_\lambda \delta^2 + \cdots; \\
    c &= c_0 + k_c \delta + n_c \delta^2 + \cdots; \\
    \gamma &= \gamma_0 + k_\gamma \delta + n_\gamma \delta^2 + \cdots,
\end{align*}
\tag{4}
$$

The power series in (4) are absolutely convergent. The rate of their convergence is directly related to the magnitude of the temperature drop $\delta$ and can be controlled by the experimenter.

An analysis of published data shows that, within the temperature ranges between phase transformations, the relative coefficients remain usually $|k_\lambda| \leq 3 \cdot 10^{-3} \text{deg}^{-1}$ and $|n_\lambda| \leq 3 \cdot 10^{-6} \text{deg}^{-2}$, making the conditions for the optimum convergence of series (4)

$$
|k_\lambda| \leq 0.1 \quad \text{and} \quad |n_\lambda| \leq 0.01
$$

easily realizable in thermophysical measurements. For instance, conditions (5) allow measurements with $\delta = 10-100^\circ \text{C}$, and with $\delta \leq 5^\circ \text{C}$ we have for series (4):

$$
|k_\lambda| \leq 0.01 \quad \text{and} \quad |n_\lambda| \leq 0.0001.
$$

If conditions (6) are satisfied, then the thermophysical parameters can be represented as linear functions of the temperature drop $\delta$.

The basic principle of a $\lambda$-calorimeter is shown schematically in Fig. 1. The temperature distribution in the active zone of this calorimeter is also indicated here.

The calorimeter consists of a metal block $B$ and a solid copper rod $C$ mounted coaxially. Between them there is a rather narrow gap filled during the experiment with the test gas. The calorimeter is heated monotonically by a heater $H$ spread uniformly over the outside surface of the block.

Generally, the heat can be transmitted through the gap by conduction, convection, and radiation. Convection can usually be eliminated from gaseous interlayers without major difficulties. The radiative component is appreciable at high temperatures, however, and must be accounted for.

The temperature drops will be referred to the rod temperature:

$$
\theta(r, \tau) = t(r, \tau) - t_e(\tau).
$$

The thermal flux through the gaseous interlayer is determined by the heat capacity $C_C(t)$ and the heating rate $b_C(\tau)$ of the rod:

$$
Q(t) = C_C(t) b_C(\tau).
$$

The thermal flux penetrates through the gap by conduction $Q_\lambda$ and by radiation $Q_r$:

$$
Q(t) = Q_\lambda(t) + Q_r(t).
$$

The following expression applies to the radiated flux $Q_r$:

$$
Q_r(t) = e_\sigma q_0 2\pi R \delta (T^4_B - T^4)\tag{10}
$$

The Fourier equation applies to the conduction flux $Q_\lambda$:

$$
Q_\lambda(t) = \lambda(t) \frac{dt}{dr}.
$$

Integrating (11) from $r = R_C$ to $r = R_B$ and retaining only the linear term in series (4) will yield

$$
\lambda(t) = \frac{Q_\lambda}{4\pi \delta_{BC}} \ln \frac{R_B}{R_C} = \frac{Q_r - Q_t}{2\pi \delta_{BC}} \ln \frac{R_B}{R_C},\tag{12}
$$

where $t = t_C + 1/2\delta_{BC}$. 

592