Concerning the heat conduction problem with a nonlinear boundary condition, as in the case of a boiling process, the effect of the time interval in the numerical solution scheme on the error of the solution is analyzed here.

The accuracy of the solution to heat conduction problems by the grid method depends on the length of time intervals ($\delta \tau$) and space intervals ($h$). The error incurred in finite-difference schemes has been estimated only qualitatively in theoretical analyses of this method of solution: its order of magnitude but not its actual value has been determined. In the practical sense, the accuracy depends not only on the size of $\delta \tau$ and $h$ but also on the characteristics of the temperature field and on the temperature drops in time as well as in space. The characteristics of the field depend on the constraints and on how the thermophysical properties vary as functions of the temperature. It will be shown here that a drastic change in the boundary conditions, as during boiling, leads to large errors in a numerical solution with improperly selected time intervals.

The mathematical model of the phenomenon under study is

$$\frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) - c_v \frac{\partial T}{\partial \tau} = 0 \quad (0 < x < R, \tau > 0), \quad (1)$$

$$T(x, 0) = T_i, \quad (2)$$

$$\alpha (T_i - T_s) + \lambda \frac{\partial T(0, \tau)}{\partial x} = 0, \quad (3a)$$

$$- \lambda \frac{\partial T(0, \tau)}{\partial x} = q(T_i) = f(T_i), \quad (3b)$$

$$\frac{\partial T(R, \tau)}{\partial x} = 0. \quad (4)$$

The problem is a nonlinear one with nonlinearities of the first kind $\lambda = \lambda(T)$, $c_v = c_v(T)$ as well as of the second kind $\alpha(T_s)$ or $q(T_s)$. Data on $\lambda(T)$ and $c_v(T)$ for the plate material (grade 08 steel) are given in the handbook [1]. We note that, within the test range of temperatures, both $\lambda(T)$ and $c_v(T)$ pass through an extremum, i.e., these temperature characteristics are by no means linear. The problem of cooling a plate with water was solved for a plate $2R = 0.16$ m thick and at an initial temperature $T_i = 1400^\circ C$, with the water at a constant temperature $T_s = 20^\circ C$. The heat-transfer coefficient $\alpha$ or the thermal flux $q$ at the surface during heat transfer through boiling attain their characteristic maximum during the critical boiling mode.

In order to obtain numerical results, we used an electrical model: a resistor grid suitable for a solution by the implicit finite-difference scheme [2]. The discreteness of the solution with respect to time and space made it easy to change the model parameters on each step, so as to account for the nonlinearities, by varying the time interval during the solution process.

In order to take into account the \( \alpha(T) \) and the \( q(T) \) characteristics when this method is used [2], the electrical resistances, which either simulate the thermal resistances \( R_\alpha \) to heat transfer at the surface or conduct current \( I_q \) simulating the thermal flux at the surface, are changed on each step of the solution in accordance with the respective surface temperature. When analyzing the effect of the boundary conditions, therefore, one may succeed equally well in simulating either the \( \alpha(T) \) or the \( q(T) \) characteristic, it is also feasible to transfer from one to the other on the same electrical model.

The choice of the method was governed by the aim of the study, namely to establish the effect of drastic changes in \( \alpha(T) \) or \( q(T) \) on the error in the solution using different time intervals. The space interval \( h \) was held constant and equal to \( R/8 \). Such an interval, without making the solution process more laborious, could yield an accuracy within the accuracy limit of the measuring circuit.* The time interval for the problem with such boundary conditions should be adjustable and its proper choice depends on \( \alpha(T) \), \( q(T) \). The consequence of an error in determining \( T_s \) would be an incorrect value of \( \alpha \) given for the respective step. With an improper (too long) time interval, the \( \alpha(T) \) characteristic (its maximum) may have become blurred or altogether lost.

Variations in the surface temperature of the plate are shown in Fig. 1 corresponding to variants of the solution with improper and fixed time intervals as listed in Table 1.

*With an appropriate choice of \( \delta t \) intervals, the error did not exceed 0.1%.