
NONSTEADY HEAT AND MASS TRANSFER IN DRYING BY REDUCED PRESSURE

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The article examines nonsteady heat and mass transfer in the process of drying by reduced pressure, on the assumption that the moisture is situated on the surface of an infinite plate.

Drying by reduced pressure may be effected in various regimes: linear reduction of moisture content, linear reduction of temperatures, or constant rate of reducing pressure. The most favorable regime is linear reduction of moisture content which makes it possible to carry out the process of moisture removal at optimum speed. In this case the moisture content has to change according to the regularity

\[ U = U_i - N_t. \] (1)

The drying rate is determined on the basis of the technological requirements that the product has to fulfill. The drying rate depends on the heat flux supplied for evaporating the liquid [1]:

\[ N = \frac{q_f}{r_m_d_m}. \] (2)

If we neglect the change of evaporation heat, of the heat-exchange surface, and of the weight of dry substance in the drying process, then the constant value of the drying rate is determined by the constant value of the heat flux. In the process of realizing this regime of drying by reduced pressure, evaporation of moisture is effected by liberation of the internal energy of the moist material [2].

If there is considerable thermal resistance or if the particles are large, the evaporation process of the moisture will to a certain extent be affected by the inhomogeneity of the temperature field inside the particles. Finding the temperature field is connected with the solution of the differential equation of heat conduction. For an infinite plate, on condition that the moisture is only on the surface, the equation is as follows:

\[ \frac{\partial T(x; \tau)}{\partial \tau} = a \frac{\partial^2 T(x; \tau)}{\partial x^2}. \] (3)
The solution of Eq. (3) with the boundary conditions

\[ T(x; 0) = T_i; \]  
\[ \pm \frac{\partial T( \pm b; \tau )}{\partial x} + \frac{q}{\lambda} = 0; \]  
\[ \frac{\partial T(0; \tau )}{\partial x} = 0 \]  

has the form [3, 4]

\[ T(x; \tau ) = T_i - \frac{q}{c_v b} \left[ x - \frac{\frac{b^2 - 3x^2}{6a} - \frac{b^2}{a} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n^2 \pi^2} \cos \left( \frac{n \pi x}{b} \right) \exp \left( -\frac{n^2 \pi^2 \sigma}{b^2} \right) \right]. \]  

With \( x = b \), Eq. (7) yields the dependence of the surface temperature on time:

\[ T(b; \tau ) = T_i - \frac{q}{c_v b} \left[ 1 + \frac{\frac{b^2}{3a} - \frac{b^2}{a} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n^2 \pi^2} \cos (n \pi) \exp \left( -\frac{n^2 \pi^2 \sigma}{b^2} \right) \right]. \]  

Assuming that the temperature of the moisture on the surface of the material is equal to the temperature of the surface of the plate, and using Antoine's equation [5], we obtain the dependence of the pressure of the medium above the surface of the plate on time that ensures a constant drying rate:

\[ P = \exp \left[ A - \frac{q}{c_v b} \left[ \tau + \frac{\frac{b^2}{3a} - \frac{b^2}{a} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n^2 \pi^2} \cos (n \pi) \exp \left( -\frac{n^2 \pi^2 \sigma}{b^2} \right) \right] \right]. \]  

Experiments showed that the deviation from the linear regularity (1) due to increased evaporation heat in proportion to reduced temperature of the evaporating liquid does not exceed 5%. The process of decreasing pressure above the surface of the material continues up to some minimum value that depends on the characteristics of the vacuum pump [6].

We assume that henceforth, beginning at the instant \( \tau_1 \) determined from Eq. (9), the pressure above the surface of the plate remains constant and equal to this minimum value. Then the temperature of the surface of the material does not decrease any more although the heat supply to the surface does not cease since the inner layers of the plate are warmer than the outer layers.

If we approximate expression (7) for \( \tau = \tau_1 \) by a parabolic dependence and determine the surface temperature from (8) for \( \tau = \tau_1 \), we obtain new boundary conditions of the differential equation (3):

\[ T(x; \tau_1) = T_0 \left( \frac{b^2 - x^2}{b^2} \right); \]  
\[ T_s(\tau_1) = T_1 - \frac{q}{c_v b} \left[ \frac{b^2}{3a} - \frac{b^2}{a} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n^2 \pi^2} \cos (n \pi) \exp \left( -\frac{n^2 \pi^2 \sigma}{b^2} \right) \right]. \]  

with which its solution has the form

\[ T(x; \tau) = T_s + \frac{32(T_0 - T_s)}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n + 1)^2} \exp \left[ -\frac{a(2n + 1)^2 \cdot \pi^2 (\tau - \tau_1)}{4b^2} \right] \cos \left( \frac{(2n + 1) \pi x}{2b} \right). \]

Figure 1 shows the results of mathematical modeling of the temperature profile over the surface of the material for different instants. The object of investigation was a polystyrene plate 0.3 × 0.4 × 0.04 m in size from whose surface isopropyl alcohol evaporated.

If we determine the temperature gradient on the surface of the plate from Eq. (12) and substitute it into the Fourier equation