NOTATION

$\overline{\alpha}$, Average heat-transfer coefficient; $F$, area of heat-transfer surface of duct; $\Delta T_{\text{log}}$, log-mean temperature difference; $Q$, quantity of heat; $T_w$, average wall temperature; $t_i$, $t_{i+1}$, thermocouple readings; $l_i$, thermocouple spacing; $k$, number of thermocouples; $Nu_0$, $\xi_0$, average Nusselt number and coefficient of hydraulic friction for smooth duct; $Nu$, $\xi$, the same for the duct with intensifier; $Re$, Reynolds number; $\alpha$, flow swirl angle; $\mu$, effective viscosity of fluid; $C_p$, specific heat at constant pressure; $\lambda$, thermal conductivity; $D$, inside diameter of duct; $d$, diameter of central stem of intensifier; $\rho$, density of fluid; $L$, length of duct.

LITERATURE CITED


COUPLED PROBLEM OF STEADY-STATE HEAT TRANSFER DURING TURBULENT FLOW OF LIQUID THROUGH A PLANE SLOT WITH DISSIPATION OF MECHANICAL ENERGY

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An analytical solution is obtained for the coupled problem of steady-state heat transfer during turbulent flow of a liquid through a plane slot with dissipation of mechanical energy.

The problem of steady-state heat transfer during turbulent flow of the coolant and the need to account for dissipation of mechanical energy in the analysis of this problem arise in practical applications [1] such as transport of crude oil under conditions of Northern climate [2].

We make the following assumptions: 1) The flow of the fluid and the heat transfer are both quasisteady; 2) the fluid is incompressible and its physical properties remain constant; 3) the change in the thermal flux density along the axis caused by heat conduction and turbulent heat transfer is small in comparison with its change in the transverse direction [1]; 4) the flow through the heat-transfer region has been hydrodynamically stabilized; 5) the temperature of the liquid and the slot wall in the entrance section is a known function of the transverse coordinate $y$; 6) the temperature of the outside surface of the slot is given as an arbitrary integrable function of the axial coordinate $x$.

The problem will be formulated as a system of equations in dimensionless variables: equation of energy for the liquid

$$
W(Y) \frac{\partial \theta_1}{\partial X} = \frac{\partial}{\partial Y} \left[ \frac{Re}{2} \left( \frac{1}{Pr} + \frac{\varepsilon_0}{v} \right) \frac{\partial \theta_1}{\partial Y} \right] + \frac{Br}{Pr} \sqrt{\frac{\varepsilon_0}{8}} \left( 1 + \frac{\varepsilon_0}{v} \right) \left( \frac{dW}{dY} \right)^2,
$$

(1)

equation of heat conduction for the wall

$0 < X < \infty$, $0 < Y < 1$. 

with the boundary conditions

\[
\begin{align*}
\theta_1 (0, Y) &= f_1 (Y), \\
\frac{\partial \theta_1 (X, 0)}{\partial Y} &= 0, \\
\theta_2 (0, Y) &= f_2 (Y), \\
\theta_2 (X, \delta) &= \varphi (X), \\
\frac{\partial \theta_2}{\partial X} \bigg|_{X=\infty} &= 0;
\end{align*}
\]

and the conditions of coupling for the temperature fields

\[
\frac{\partial \theta_1 (X, 1)}{\partial Y} = K_x \frac{\partial \theta_2 (X, 1)}{\partial Y}, \quad \Theta_1 (X, 1) = \Theta_2 (X, 1) = \chi (X).
\]

We let function \( \phi (X) \) reach a finite value at \( X \to \infty \) and let the unknown function at the coupling boundary be the temperature at that boundary, assuming also that \( N_{Pr, turb} = 1 \).

For channels of simple geometrical shapes there are several semiempirical relations available which describe the velocity profile and yield the coefficients of turbulent momentum transfer. In this study will be used the Pey relations for the velocity profile \( W (Y) \) and the coefficient of turbulent momentum transfer \( \varepsilon_\theta / \nu \).

The problem (1)-(8) can be solved by the method of finite differences [4, 5] or an exact analytical solution can be obtained [6], but the approximate analytical solution proposed here is more convenient for an analysis of coupled heat-transfer processes and engineering calculations.

The solution to the system of Eqs. (1)-(8) will be sought as the sum of two functions

\[
\begin{align*}
\Theta_1 (X, Y) &= \Theta_{1W} (Y) + \Theta_{1X} (X, Y), \\
\Theta_2 (X, Y) &= \Theta_{2W} (Y) + \Theta_{2X} (X, Y).
\end{align*}
\]

For determining these functions we obtain from Eqs. (9) and (10) four interdependent equations with corresponding boundary conditions

\[
\begin{align*}
\frac{d}{dY} \left[ \frac{N_{Re}}{2 \delta_0} \left( \frac{1}{Pr} + \frac{\varepsilon_\theta}{\nu} \right) \frac{\partial \Theta_{1W}}{\partial Y} \right] + \frac{N_{Re}}{\delta_0} \left[ \frac{\varepsilon_\theta}{\nu} \right] (1 + \frac{\varepsilon_\theta}{\nu}) \left( \frac{dW}{dY} \right)^2 &= 0, \\
\frac{d^2 \Theta_{2W}}{dY^2} &= 0, \\
\Theta_{2W} (\delta) &= \varphi (\infty), \quad \Theta_{2W} (1) = \Theta_{2W} (1) = \chi_w, \quad \frac{\partial \Theta_{2W} (1)}{\partial Y} = K_x \frac{\partial \Theta_{2W} (1)}{\partial Y}; \\
W (Y) \frac{\partial \Theta_{1X}}{\partial X} &= \frac{\partial}{\partial Y} \left[ \frac{N_{Re}}{2 \delta_0} \left( \frac{1}{Pr} + \frac{\varepsilon_\theta}{\nu} \right) \frac{\partial \Theta_{1X}}{\partial Y} \right], \\
\Theta_{1X} (0, Y) &= f_1 - \Theta_{1W} (Y), \quad \frac{\partial \Theta_{1X} (X, 0)}{\partial Y} = 0; \\
\frac{4}{N_{Re}} \frac{\partial \Theta_{2X}}{\partial X^2} + \frac{\partial^2 \Theta_{2X}}{\partial Y^2} &= 0, \\
\Theta_{2X} (0, Y) &= f_2 - \Theta_{2W} (Y), \quad \Theta_{2X} (X, \delta) = \varphi (X) - \Theta_{2W} (Y), \\
\Theta_{1X} (X, 1) &= \Theta_{2X} (X, 1) = \chi (X) - \chi_w = \chi_t (X),
\end{align*}
\]

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