EVALUATING THE HEAT PROOFING PROPERTIES
OF ANISOTROPIC INSULATION

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The thermal resistance of vacuum-shield insulation is evaluated on the basis of earlier test
data [1] on the anisotropy of heat conduction through it.

The data in [1] indicate a very pronounced anisotropy of heat conduction through a vacuum-shield thermal insulation along and across its layers. For this reason, an evaluation of the thermal resistance of such an insulation must take into account this anisotropy as well as the specific structural conditions under which it is used.

We will consider the case of steady-state heat transmission through a flat layer of anisotropic vacuum-shield insulation with heat transfer at its end surfaces. This case is typical of many practical structural designs involving the installation of vacuum-shield insulation.

The schematic diagram in Fig. 1 shows the transverse section through a flat layer of anisotropic insulation having the shape of an infinitely long prism. The heat transfer at the end surfaces is defined by boundary conditions of the third kind.

Let the constant thermal conductivities along the x- and y-axis be \( \lambda_x \) and \( \lambda_y \), respectively, and the temperature of the insulation be \( T \). The differential equation of heat conduction in this case will be

\[
\lambda_x \frac{\partial^2 T}{\partial x^2} + \lambda_y \frac{\partial^2 T}{\partial y^2} = 0. \tag{1}
\]

and the boundary conditions:

- for \( y = b \) \( \lambda_y \frac{\partial T}{\partial y} = \alpha_1 (T_1 - T) \);
- for \( y = 0 \) \( \lambda_y \frac{\partial T}{\partial y} = \alpha_1 (T - T_1) \);
- for \( x = 0 \) \( \lambda_x \frac{\partial T}{\partial x} = \alpha_1 (T - T_2) \);
- for \( x = a \) \( \lambda_x \frac{\partial T}{\partial x} = \alpha_1 (T_2 - T) \).

In the dimensionless coordinates

\[
\xi = \frac{x}{b} \sqrt{\frac{\lambda_b}{\lambda_x}} \quad \text{and} \quad \eta = \frac{y}{b},
\]

Eq. (1) becomes

\[
\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \eta^2} = 0. \tag{2}
\]

We introduce the following notation:

\[ \frac{\lambda_x}{\lambda_y} = k; \quad R_y = \frac{b}{\lambda_y}; \quad R_1 = \frac{1}{\alpha_1}; \quad R_2 = \frac{1}{\alpha_2}. \]

Then the boundary conditions can be rewritten as

\[ \frac{\partial T}{\partial \eta} = \frac{R_y}{R_1} (T_1 - T); \quad \text{for } \eta = 1 \]
\[ \frac{\partial T}{\partial \eta} = \frac{R_y}{R_2} (T - T_2); \quad \text{for } \eta = 0 \]
\[ \frac{\partial T}{\partial \xi} = \frac{R_y}{R_1} \frac{1}{k} (T - T_1); \quad \text{for } \xi = 0 \]
\[ \frac{\partial T}{\partial \xi} = \frac{a}{b \sqrt{k}} \frac{R_y}{R_3} \frac{1}{k} (T_2 - T); \quad \text{for } \xi = \beta k \]

The solution will be sought in the form

\[ T = T_0 + f(\xi, \eta), \]

with \( T_0 \) corresponding to a one-dimensional temperature field (disregarding the heat transfer at the end surface of the specimen).

If we let \( T_0 = C_0 + C_1 \eta \), then the boundary conditions will yield

\[ C_1 = \frac{R_y}{R_1} (T_1 - C_0 - C_1); \quad C_1 = \frac{R_y}{R_2} (C_0 - T_2), \]

from where

\[ C_0 - T_2 = \frac{R_y}{R_1} (T_1 - C_0) - \frac{R_y}{R_1} (C_0 - T_2). \]

We add the following notation:

\[ C_0 - T_2 = C_0'; \quad T_1 - T_2 = \Delta T, \]

so that

\[ C_0' = \frac{R_y \Delta T}{R_y + R_1 + R_2}; \quad C_1 = \frac{R_y \Delta T}{R_y + R_1 + R_2}. \]

Now Eq. (2) becomes

\[ \frac{\partial^2 f}{\partial \xi^2} + \frac{\partial^2 f}{\partial \eta^2} = 0 \quad (3) \]

and the boundary conditions:

\[ \text{for } \eta = 1 \quad \frac{\partial f}{\partial \eta} = -\frac{R_y}{R_2} f; \quad (4) \]