A method is described of simulating a transient heat-conduction process on a general-purpose analog computer. For illustration, the problem of optimally heating a polymer film has been solved by this method.

The transmission of thermal energy through various kinds of walls is in many industrial applications effected by a transient mode of heat conduction. Thus, there arise problems of optimization.

One very often encounters problems in temperature regulation, which in many ways determines the product quality, and reduces them to that of simulating the object through which transient heat conduction takes place while it interacts with the control device. The simulation process reveals the optimum regulation modes and determines the optimum parameters of the automatic control system (ACS). Such problems are solved with the aid of analog computers (AC) [1, 2].

Analog-computer simulation of general transient heat-conduction processes and the necessity of considering a three-dimensional temperature field give rise to various difficulties [3] involved with the analog representation of partial differential equations with variable boundary conditions and with the resulting transcendental transfer functions dependent on the boundary conditions of a given problem. In this study we will consider one possible method of overcoming these difficulties.

We consider the problem of transient heat conduction through an infinitely large plane wall:

\[
\frac{\partial \theta (\xi; Fo)}{\partial Fo} = \frac{\partial^2 \theta (\xi; Fo)}{\partial \xi^2}, \quad Fo > 0; \ 0 < \xi < 1. \tag{1}
\]

with the initial and the boundary conditions

\[
\theta (\xi; 0) = 0, \tag{2}
\]
\[
\frac{\partial \theta (0; Fo)}{\partial \xi} + q_1(Fo) = 0, \tag{3}
\]
\[
\frac{\partial \theta (1; Fo)}{\partial \xi} - q_2(Fo) = 0. \tag{4}
\]

Quantities \( q_1 \) and \( q_2 \) in Eqs. (3) and (4) are the referred thermal fluxes on the wall surfaces:

\[
q_1(Fo) = \frac{l}{\lambda} q_{11}(Fo) - B_1 [\theta (0; Fo) - \theta_1(Fo)], \tag{5}
\]
\[
q_2(Fo) = \frac{l}{\lambda} q_{12}(Fo) - B_2 [\theta (1; Fo) - \theta_2(Fo)]. \tag{6}
\]

We apply the Laplace transformation to Eqs. (1)–(6) with respect to \( Fo \) [4]. The transform functions will be denoted by capital letters. Solving the transform equations with the constraints will yield

\[
\Theta (\xi; p) = W_1(\xi; p) Q_1(p) + W_2(\xi; p) Q_2(p), \tag{7}
\]
Let us now examine the structure of functions $Q_1$ and $Q_2$:

$$Q_1 (p) = \frac{1}{\lambda} Q_{11} (p) - B_{11} \left[W_1 (0; p) Q_1 (p) + W_2 (0; p) Q_2 (p) - \Theta_1 (p) \right].$$

For $Q_2$ we have, analogously,

$$Q_2 (p) = \frac{1}{\lambda} Q_{12} (p) - B_{12} \left[W_1 (1; p) Q_1 (p) + W_2 (1; p) Q_2 (p) - \Theta_2 (p) \right].$$

On the basis of Eqs. (7), (11), and (12), we design a structural diagram (Fig. 1) of a model which combines operators and transform functions in the form (8) and (9).

The form of the transfer functions does not depend on the specific conditions of the problem and is determined only by the wall geometry.

The structure of a model depends on the boundary conditions. Boundary conditions of the third kind result in feedback coupling at the surfaces and in transfer functions representing the interaction between processes at these surfaces.

In order to make an analog-computer simulation of the process feasible, we have expanded the transcendental transfer functions into series with respect to their poles:

$$W_1 (\xi; p) = \frac{2}{p} + 2 \sum_{n=1}^{\infty} \frac{1}{n^2 \pi^2} \cos n \pi \xi,$$

$$W_2 (\xi; p) = \frac{2}{p} + 2 \sum_{n=1}^{\infty} (-1)^n \frac{n^2 \pi^2}{n^2 \pi^2 - p + 1} \cos n \pi \xi. \quad (14)$$