A general approach is considered for the calculation of the temperature field of a ventilated electronic instrument of cassette construction through the reduction of its heated zone to a uniform body. An approximate solution of the problem is proposed and the results of calculations are compared with experimental data.

Ventilated electronic instruments of cassette construction are widely used in technology. The search for their optimum construction is connected with the analysis of the thermal conditions of the instrument, which necessitates the improvement of the methods of calculating their temperature fields. The thermal model of an instrument of cassette construction can be represented in the form of a uniform parallelepiped with distributions of heat sources and sinks. The role of the latter is filled by gas streams passing through and carrying with them part of the energy from the instrument. A basis for this model as well as its mathematical description in the form of a system of equations for the temperature fields of the cassettes and the interlayers of gas between them are presented in [1] for the case of natural ventilation. The same system of equations will clearly also be valid for the forced ventilation of an instrument. The difference will consist only in the means of determining the flow rate of gas through the instrument: with natural ventilation additional equations are needed to calculate the gas flow rate while for forced ventilation the flow rate is assumed to be known.

The temperature field is described by the system of equations [1]

\[
\lambda_1 \frac{\partial^2 \theta}{\partial x^2} + \lambda_2 \frac{\partial^2 \theta}{\partial y^2} = \alpha_v (\theta - \bar{\theta}_a) - q_v, \tag{1a}
\]

\[
\bar{\theta} = \bar{\theta}_a - \frac{\omega}{\alpha_v} \frac{d\bar{\theta}_a}{dx}, \quad \omega = \frac{c_p G}{b (h + \delta)} \tag{1b}
\]

and the boundary conditions:

\[
\left[ \frac{\partial \theta}{\partial x} - \frac{\alpha_{x1}}{\lambda_1} \bar{\theta} \right]_{x=0} = 0, \quad \left[ \frac{\partial \theta}{\partial x} + \frac{\alpha_{x2}}{\lambda_1} \bar{\theta} \right]_{x=L} = 0, \tag{2}
\]

\[
\left[ \frac{\partial \theta}{\partial y} - \frac{\alpha_{y1}}{\lambda_2} \bar{\theta} \right]_{y=-a} = 0, \quad \left[ \frac{\partial \theta}{\partial y} + \frac{\alpha_{y2}}{\lambda_2} \bar{\theta} \right]_{y=a} = 0, \tag{3}
\]

\[
\bar{\theta} \big|_{x=L} = \bar{\theta}_{in}. \tag{4}
\]

In Eqs. (1) the volumetric coefficient of convective heat exchange \( \alpha_v \) is connected with the local coefficient of heat exchange \( \alpha \) by the dependence [1]

\[
\alpha_v = \frac{2\alpha}{h + \delta}. 
\]
The coordinate system is set up as shown in Fig. 1. It is assumed that the temperature variation along the width of the cassettes (along the z axis) is insignificant. The latter assumption is not necessary in principle and is adopted mainly to reduce the calculations. Moreover, in real instruments with forced ventilation the important temperature variation occurs in the direction of air movement (along the x axis) and perpendicular to the plane of the cassettes (along the y axis).

The values \( \alpha_x \) and \( \alpha_y \) in (2) and (3) allow for the heat exchange with the surrounding medium from the ends of the cassettes and from the faces of the heated zone which are perpendicular to the y axis, respectively. In general the conditions (2) and (3) can reflect the presence of conductive heat sinks at the boundaries of the heated zone of the instrument.

We will seek an approximate solution of the problem by the method of successive averaging of the unknown function [2, 3].

Let us introduce the averaging operator \( I_x \)

\[
I_x [f] = \frac{1}{l} \int_0^l f(x) \, dx
\]

and designate

\[
I_x [\theta] = \tilde{\theta}, \quad I_x [\theta_a] = \bar{\theta}_a, \quad I_x [q_x] = \overline{q}_x,
\]

\[
\tilde{\theta} \big|_{x=a} = \theta_{\text{out}}, \quad \tilde{\theta} \big|_{x=b} = \theta_{\text{in}}.
\]

We apply the operator \( I_x \) to all the terms of Eqs. (1), allowing for the conditions (2) and (4), and then combine the results of the termwise action of the operator

\[
\lambda_x \frac{d^2 \tilde{\theta}}{dy^2} = \frac{\alpha_{x1}}{l} \theta_a + \frac{\alpha_{x2}}{l} \theta_y - \overline{\alpha} (\bar{\theta} - \bar{\theta}_a) - \overline{q}_x,
\]

(5)

Here \( \overline{\alpha}_y \) is the averaged value of the volumetric coefficient of convective heat exchange, for the calculation of which one must know the coefficients of convective heat exchange \( \overline{\alpha}_y \) in the channels divided by the mean integral temperature difference:

\[
\overline{\alpha}_y = \overline{\alpha}_y (y) = \frac{2\overline{\alpha}}{h + 2\delta}, \quad \overline{\alpha} = \frac{1}{l} \int_0^l (\theta - \theta_a) \, dx
\]

(6)

Let us adopt the specific assumptions for the method of averaging [2, 3]

\[
\frac{\theta_{\text{out}} - \theta_{\text{in}}}{\theta_a} = \psi \mp f(y), \quad \frac{\theta_a}{\theta} = \psi \pm f_a(y), \quad \frac{\theta_{\text{out}} - \theta_{\text{in}}}{\theta_a} = \psi \mp f_a(y).
\]

(7)

Then (5) can be rewritten in the following form:

\[
\lambda_x \frac{d^2 \tilde{\theta}}{dy^2} - \frac{\psi \alpha_{x1} + \psi \alpha_{x2}}{l} \tilde{\theta} = \overline{\alpha} (\bar{\theta} - \bar{\theta}_a) - \overline{q}_x,
\]

(8a)

\[
\tilde{\theta} = \tilde{\theta}_a + \frac{\psi_0}{\alpha_{x1} l} \tilde{\theta}_a.
\]

(8b)

We find the value of \( \tilde{\theta}_a \) from (8b) and substitute it into (8a):

\[
\frac{d^2 \tilde{\theta}}{dy^2} - \frac{\psi_0}{\lambda_x} \tilde{\theta} = \frac{\alpha_{x1}}{\lambda_x} \psi_0 \frac{\alpha_{x2}}{\psi_0 + \alpha_{x1} l} = \frac{\psi_0 \alpha_{x1} - \psi \alpha_{x2}}{\lambda_x l}.
\]

(9)