NUMERICAL INVESTIGATION OF INDUCTION STAGE
OF DEVELOPMENT OF NATURAL CONVECTION

É. A. Shtessel', K. V. Pribytkova, and A. G. Merzhanov

The process of development of convection in a two-dimensional region of square cross section, which is heated from below, is investigated numerically. The results are compared with earlier experimental results.

The process of development of natural convection in a two-dimensional fluid layer was investigated in [1] based on the measurements of nonstationary fields. It was shown that one of the characteristic features of this process is the existence of a period $\tau$ of the induction of convection. The experimental results obtained in [1] were generalized in the form of the following formula:

$$\tau = 70 \frac{Ra^{2.3} Pr^{-1/6}}{Ra} \quad \text{for} \quad Pr > 10 \quad \text{and} \quad Ra > 10^4.$$  

In the present article the process of development of convection is investigated numerically and the results are compared with the experimental results.

We investigated a region of square cross section filled with an inert fluid and included between two horizontal plates ($y = 0$ and $y = h$), held at constant temperatures $T_1$ and $T_0$ respectively with $T_1 > T_0$. The vertical plates ($x = 0$, $x = h$) are thermally insulated. At the initial time the temperature of the fluid is $T_0$.

The nondimensional equations describing the process have the following form:

$$\frac{\partial \vec{v}}{\partial \tau} + Pr \nabla^2 \vec{v} = -\vec{v} \cdot \nabla \vec{v} - Pr \nabla \cdot \vec{v} + Ra \vec{\theta} i;$$

$$\frac{\partial \vec{\theta}}{\partial \tau} + Pr \nabla^2 \vec{\theta} = \Delta \vec{\theta}; \quad \vec{v} = 0. \quad (2)$$

The initial conditions are:

$$\theta(x, z, 0) = \vec{v}(x, z, 0) = 0; \quad (3a)$$

![Fig. 1. Isotherms and stream functions (Ra = 1.7 \cdot 10^4, Pr = 20): a) $\tau = 1.2 \cdot 10^{-2}$; b) $2.9 \cdot 10^{-2}$; c) $1 \cdot 10^{-4}$.](image)


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and the boundary conditions are:
\[ \xi = 0, \quad \theta = 1, \quad \bar{v} = 0, \quad \xi = 1, \quad \theta = 0, \quad \bar{v} = 0, \quad \eta = 0: \frac{\partial \theta}{\partial \eta} = 0, \quad \bar{v} = 0. \]
\equal{3b}

the coordinate \( \xi = y/h \) is directed against the gravitational force.

The following quantities are chosen as the scales of distance, time, velocity, pressure, and temperature:
\[ h, \frac{a}{h^2}, \quad \frac{v}{h}, \quad \frac{\rho v a}{h^2}, \quad (T_1 - T_0). \]

The system of equations (2)-(3a, b) was solved by the method discussed in [2]. Rayleigh's number was varied in the range \( 10^3 \) to \( 5 \times 10^4 \). Prandtl's number was kept constant in the computations and was equal to 20.

The pattern of development of convection is shown in Fig. 1 for \( Ra = 1.7 \times 10^4 \). Since the pattern is symmetric, isolines of the stream function are shown on the left and isotherms on the right. In the beginning at the bottom surface of the fluid, the convection motion appears in the form of two vortices rotating in opposite directions. The values of the stream functions increase with time and the vortices move in the direction opposite to the force of gravity and occupy the entire volume of the fluid. A two-vortex cell is formed; at the center of this cell, the fluid rises and subsides at the thermally insulated edges. The increase in the magnitude of the stream functions and the extension of the vortices to the entire volume of the fluid is sufficiently rapid, only over a certain time interval, i.e., the induction period of the convection. The magnitude of the stream function and the nature of development of convection depend on the Rayleigh number.

![Fig. 2. Dependence of \( \theta \) on \( \tau \) for different \( \xi \): 1) \( \xi = 0.1 \); 2) 0.2; 3) 0.3; 4) 0.5; 5) 0.7; 6) 0.8 and dependence of \( |\psi_m| \) on \( \tau \).

![Fig. 3. Dependence of \( \phi \) on \( \xi \) for \( \eta = 0.5 \) for different instants of time \( \tau \): 1) \( \tau = 2 \times 10^{-2} \); 2) \( 3.5 \times 10^{-2} \); 3) \( 5 \times 10^{-2} \); 4) \( 1.25 \times 10^{-1} \); 5) \( 4 \times 10^{-1} \) and the dependence of \( \phi \) on time for different \( \xi \): 1) \( \xi = 0.3 \); 2) 0.5; 3) 0.7; 4) 0.8.]