We present a method for electrically modelling nonlinear contact heat-transfer problems both with and without taking into account the thermal conductivity of the contact layer.

To determine the temperature fields in bodies of complex configurations, as well as in the solution of other problems of field theory with complicated boundary conditions, wide use is being made of the method of electrical analogy to solve successfully both linear and nonlinear problems. In particular, analog methods are available for solving nonlinear heat conduction with boundary conditions of the types I-IV on ohmic resistance networks (R-networks) [1], based on Liebmann's method, the solution being obtained iteratively. References [2, 3] are directly concerned with modelling nonlinear problems of contact heat transfer.

The method of successive approximations is applied in [2] also, however, in contrast to the procedure in [1], only the boundary resistances are changed after each approximation, and not all the resistance of the R-network as was required in [1].

The solution given in [3] involved a linearization of the boundary conditions, followed by an introduction of new functions and a reassignment of the remaining boundary conditions. This method requires matching of the separate resistances of the conducting media employed for modelling the bodies in contact (or a corresponding selection of the parameters of the R-networks used in modelling with networks of ohmic resistances). The solution of nonlinear problems of field theory by RC-networks was, until recently, considered to be unrealizable in general, being possible only as a result of applying special transformations and using special devices to model nonlinear boundary conditions [4]. Unfortunately, the problem with boundary conditions of the fourth kind remained unsolved in view of its complexity and the need for a special approach.

In this paper we present a method for modelling a nonlinear contact heat-transfer problem, based on a combined use of passive models and apparatus constructed on the principle of electronic modelling.

Since the apparatus for modelling contact heat transfer, which is our basic concern here, are universal, i.e., they are equally available for solving both stationary and nonstationary problems, for simplicity we can, with no loss in generality, consider the stationary problem.

Assume that the thermal conductivity coefficients of two bodies in contact are functions of the temperature: \( \lambda_1(t) \) and \( \lambda_2(t) \).

Then the stationary heat conduction equations for these bodies may be written as follows:

\[
\begin{align*}
\frac{\partial}{\partial x} \left[ \lambda_1(t) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \lambda_1(t) \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \lambda_1(t) \frac{\partial T}{\partial z} \right] &= 0, \\
\frac{\partial}{\partial x} \left[ \lambda_2(t) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \lambda_2(t) \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \lambda_2(t) \frac{\partial T}{\partial z} \right] &= 0.
\end{align*}
\]

(1)
Boundary conditions of the fourth kind, which equate temperatures and heat fluxes on the boundary, are usually written in the form:

\[
\frac{\lambda_1(t)}{\partial n} \bigg|_1 = \frac{\lambda_2(t)}{\partial n} \bigg|_2, \quad (2)
\]
\[
t_1 = t_2. \quad (3)
\]

However, in recent years the concept of contact heat transfer has been somewhat broadened (see, for example, [3]) to take into account the thermal conductivity of the contact layer formed of protrusions of roughness. In this case, the boundary condition (2) stays the same and the condition (3) is replaced by the following:

\[
k(t_1 - t_2) = -\lambda_1(t) \frac{\partial t}{\partial n}. \quad (4)
\]

We apply to Eqs. (1)-(4) the integral transformations

\[
\Phi = \int \lambda_1(t) \, dt; \quad F = \int \lambda_2(t) \, dt. \quad (5)
\]

The Eqs. (1) are thereby converted into Laplace equations, which may be simulated by passive models (R-networks or electrically conducting paper).

The boundary conditions (2)-(4) assume the form:

\[
\frac{\partial \Phi}{\partial n} \bigg|_1 = \frac{\partial F}{\partial n} \bigg|_2, \quad (6)
\]
\[
t_1(\Phi) = t_2(F). \quad (7)
\]
\[
k \left[ t_1(\Phi) - t_2(F) \right] = -\frac{\partial \Phi}{\partial n} \bigg|_1. \quad (8)
\]

The electrical arrangement for the case of the boundary conditions (6)-(7) is shown in Fig. 1.

Between the boundary points of the two passive models 1 and 2 there is included a rheostat 3, which, along with the servomotor 4, the differential amplifier 5, and the two functional transformations 6 and 7, defines the following system. Control or variation of the resistance 3 proceeds so long as the equality (7) is not achieved, i.e., as long as the error signal put out by the differential amplifier is not equal to zero.

Since the functions \(\lambda_1(t)\) and \(\lambda_2(t)\) are, in general, distinct, the functions \(\Phi\) and \(F\) may then so differ from one another that an alternate situation arises, wherein the current must be a flow from a point with a smaller potential to a point with a higher potential. This situation will prevail, for example, for a general direction of the current from model 1 to model 2, the function \(F\) on the boundary being larger than the

![Fig. 1. Arrangement for achieving boundary conditions of the fourth kind.](image)