A calculation is presented of the temperature distribution in a laminar liquid flow moving in a circular tube. The heat exchange on the outer surface of the channel is determined by the Stefan–Boltzmann law.

Let us investigate the transfer of heat inside a liquid flowing in a circular pipe whose wall has zero resistance. The local heat-flux density on the channel wall is proportional to the difference between the fourth powers of the temperatures of the outside surface and the gaseous medium. Assuming the liquid flow to be stabilized with a parabolic velocity profile, we consider the two cases of greatest practical interest:

1) radiative heating of a gaseous medium with temperature $T_m$;
2) radiative cooling in a medium of zero temperature.

The mathematical formulation of the first problem includes the energy equation

$$\frac{\partial^2 \theta(R, X)}{\partial R^2} + \frac{1}{R} \frac{\partial \theta(R, X)}{\partial R} = (1 - R^2) \frac{\partial \theta(R, X)}{\partial X}$$

and the boundary conditions

$$\theta(R, 0) = \theta_0,$$

$$\frac{\partial \theta(1, X)}{\partial R} = K_1 [1 - \theta^4(1, X)],$$

$$\frac{\partial \theta (0, X)}{\partial R} = 0,$$

where

$$R = \frac{r}{r_o}; \ X = \frac{2x}{\rho \epsilon d_0}; \ Pe = \frac{\rho d_0}{\alpha}; \ d_o = 2r_o; \ \theta = \frac{T}{T_m}; \ K_1 = \frac{\sigma r_m r_0^2}{\lambda} \frac{d}{d_0}.$$

A general analytic method of solving heat-conduction problems with different nonlinear boundary conditions was developed in [1], and was subsequently used to calculate nonstationary radiant heating and cooling of solids [2, 3]. With suitable modifications, this method can be easily generalized to include cases of heat exchange with a flowing liquid.

Thus, the application of the transformation

$$\frac{\ln \theta}{-\rho} = \int_0^\theta \frac{d\theta}{1 - \theta^4} = \frac{1}{2} (\text{Arth} \theta + \arclg \theta)$$

to the problem (1)–(4) yields

$$\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R} \frac{\partial \theta}{\partial R} = (1 - R^2) \frac{\partial \theta}{\partial X} + \rho \delta \left( \frac{\partial \theta}{\partial R} \right)^2 \left( \rho - 4\theta \right),$$


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Fig. 1. Variation of the temperature $\theta(0, X)$ in a liquid stream (the curves represent the computer data and the points calculations by the proposed procedure): 1-3) $K_i = 0.5, 1.0,$ and $1.5,$ respectively.

To solve the system (6)-(9) it is necessary first to maintain the nonlinear function $f_1(p, \delta, \theta, \partial \theta / \partial R)$ in the right-hand side of the transformed energy equation (6). The condition $f_1 \to 0$ can be realized in the following manner, as indicated in [1, 2]: the region of variation $\theta(R, X)$ is broken up into several segments $(\theta_0 - \theta_1, \ldots, \theta_{i-1} - \theta_i, \ldots)$, in each of which it is assumed that $p_1 = 4\theta_i$. It should be noted that in the case of moderate heating (not too large values of the $K_i$ number), minimization is ensured by a sufficiently simple choice of the parameter $p$ without subdividing the entire range of temperature variation into intervals:

$$p = 4 \left( \frac{\theta_i + 1}{2} \right)^3. \quad (10)$$

The solution of the linearized equation (6) under the boundary conditions (7)-(9) is

$$\theta = \bar{\theta}_0 \sum_{n=0}^{\infty} A_n \psi_n (R) \exp \left( -2 \frac{\pi^2}{P_c} \frac{1}{d_0} \frac{x}{d_0} \right), \quad (11)$$

where $A_n$, $\psi_n$, and $\epsilon_n$ are defined in [4].

Substitution of (11) into the transformation (5), which is tabulated in [2], yields the final solution of this problem.

The process of radiative cooling in a medium of zero temperature is described by the energy equation (1) with boundary conditions

$$\theta(R, 0) = 1, \quad \frac{\partial \theta(1, X)}{\partial R} = -K_i \theta(1, X), \quad \frac{\partial \theta(0, X)}{\partial R} = 0.$$

Here

$$\theta = \frac{T}{T_0}, \quad K_i = \frac{\sigma T^4}{\lambda} \cdot \frac{d}{d_0}.$$

The transformation

$$\Phi(R, X) = \exp \left[ -\frac{P}{3} \theta^{-\frac{3}{2}} (R, X) \right], \quad (12)$$

without changing the symmetry conditions, linearizes the boundary condition

$$\frac{\partial \Phi(1, X)}{\partial R} = -p K_i \Phi(1, X).$$

This gives rise to the following nonlinear function in the transformed equation for the energy:

$$f_1 = \frac{\partial \theta}{\partial R} \cdot \frac{\partial \theta}{\partial R} \theta^{-4} (p - 4\theta) \quad (13)$$

and

$$\bar{\theta}(R, 0) = \exp \left( -\frac{P}{3} \right) = \theta_0.$$

The calculation then follows the same sequence as in the first problem. The nonlinear complex (13) is first linearized by assuming $P_1 = 4\theta_i^3$ in each interval $1 - \theta_1, \ldots, \theta_{i-1} - \theta_i, \ldots$. This is followed by the use of a solution of the type (11) and the transformation (12).