THE EFFECT OF MOTION ENERGY DISSIPATION ON
HEAT TRANSFER FOR LAMINAR FLOW OF NEWTONIAN
FLUIDS IN CIRCULAR PIPES

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An analytic solution is obtained for the problem of heat transfer in the laminar flow of
Newtonian fluids in circular pipes taking account of motion energy dissipation.

In many engineering applications it is extremely important to take account of the effect of dissipative
heating on heat transfer in the laminar flow of Newtonian fluids in pipes. But up to the present time the
study of these problems has not been sufficiently explicit; in particular there are no data which make it pos-
sible to give not only a qualitative but also a quantitative estimate of the effect of the dissipative factor on
the fundamental characteristics of heat transfer.

The aim of this paper is to fill this gap to some extent. We consider heat transfer in the steady mo-
tion of Newtonian fluids in a circular pipe when the friction curves can be approximated by a rheological
power law

\[ \tau = k \eta^n. \]  

(1)

When the velocity profile is established, the heat flux along the axis of the pipe is small by comparison
with the heat flux radially, the physical properties of the fluid are constant and there are no additional internal
heat sources, the energy equation can be written

\[ \rho c_p \frac{dT}{dx} = \frac{\lambda}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \Phi_d(r). \]  

(2)

From the equation of motion, neglecting body forces, and taking note of [1], we obtain a nondimension-
al expression for the velocity distribution and the dissipation functions:

\[ u(\eta) = \frac{m+3}{m+1} \sqrt{1 - \eta^{n+1}}, \]  

(3)

\[ \Phi_d(\eta) = \frac{m+3}{2} \eta^{n+1}. \]  

(4)

In nondimensional coordinates Eq. (2) has the form

\[ (1 - \eta^{n+1}) \frac{\partial \theta}{\partial \xi} = \frac{1}{\eta} \frac{\partial}{\partial \eta} \left( \eta \frac{\partial \theta}{\partial \eta} \right) + \frac{m+3}{2} \eta^{n+1} \]  

(5)

with the boundary conditions

\[ \theta(0, \eta) = \Theta_0; \quad \theta(\xi, 1) = 0; \quad \frac{\partial \theta(\xi, 0)}{\partial \eta} = 0. \]  

(6)

This problem was considered in the above formulation in [1] for the particular case when the fluid
temperature at the beginning of the pipe was equal to the wall temperature. In addition a solution was ob-
tained in a form which prevented precise clarification of the effect of dissipation energy on the fundamental
characteristics of heat transfer. The general solution only gave finite results for \( n = 0.5 \) (m = 2).

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TABLE 1. Values of $\text{Nu}_\infty$ for Various $m$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\text{Nu}_\infty$ ($\beta=0$)</th>
<th>$\text{Nu}_\infty$ ($\beta=1$)</th>
<th>$m$</th>
<th>$\text{Nu}_\infty$ ($\beta=0$)</th>
<th>$\text{Nu}_\infty$ ($\beta=1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.6567</td>
<td>9.6000</td>
<td>6</td>
<td>4.6192</td>
<td>19.8000</td>
</tr>
<tr>
<td>2</td>
<td>3.9494</td>
<td>11.6667</td>
<td>7</td>
<td>4.7193</td>
<td>21.8182</td>
</tr>
<tr>
<td>3</td>
<td>4.1754</td>
<td>13.7143</td>
<td>8</td>
<td>4.8044</td>
<td>23.8333</td>
</tr>
<tr>
<td>4</td>
<td>4.3544</td>
<td>15.7500</td>
<td>9</td>
<td>4.8771</td>
<td>25.8461</td>
</tr>
<tr>
<td>5</td>
<td>4.4995</td>
<td>17.7778</td>
<td>10</td>
<td>4.9402</td>
<td>27.8571</td>
</tr>
</tbody>
</table>

An acceptable method of solution was given in [2]. Using the eigenvalues and eigenfunctions given in [3] the author has qualitatively analyzed the effect of internal heat generation for $n = 1/3$.

In neither [1] or [2] was the most interesting region near to the beginning of the pipe investigated, since when the number of eigenvalues is restricted (in [3] only the first three eigenvalues were given) large errors occur in the solution for this region.

A larger region was considered in [4] but heat transfer was only discussed for a uniform distribution of internal heat sources across a section of the flow.

We seek a solution of Eq. (5) as the sum of two functions

$$
\Phi(\xi, \eta) = \Phi_1(\eta) + \Phi_2(\xi, \eta),
$$

where $\Phi_1(\eta)$ is the temperature in a stabilized segment of heat transfer satisfying the equation

$$
\frac{1}{\eta} \frac{d}{d\eta} \left( \eta \frac{d\Phi_1}{d\eta} \right) + \frac{m+3}{2} \eta^{m+1} = 0 \tag{8}
$$

with the boundary conditions

$$
\frac{d\Phi_1}{d\eta} \bigg|_{\eta=1} = 0.
$$

The function $\Phi_2(\xi, \eta)$ satisfies the equation

$$
(1 - \eta^{m+1}) \frac{\partial \Phi_2}{\partial \xi} = \frac{1}{\eta} \frac{\partial}{\partial \eta} \left( \eta \frac{\partial \Phi_2}{\partial \eta} \right) \tag{9}
$$

with the boundary conditions

$$
\Phi_2(0, \eta) = \Phi_0 - \Phi_1; \quad \Phi_2(\xi, 1) = \frac{\partial \Phi_2(\xi, 0)}{\partial \eta} = 0.
$$