TEMPERATURE DISTRIBUTION IN A REPEATEDLY HEATED POLYMER

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The conduction problem is presented and solved for a hollow two-layer cylinder having the first layer made of a polymer; the internal boundary moves and there is repeated heating from within. Allowance is made for the heat of reaction in the first layer.

We consider repeated heating from within for a hollow two-layer cylinder whose first layer is of polymer and whose second layer is highly conducting (steel, copper, etc.). Convection and radiation are responsible for the heat transfer between the polymer and the hot gas on the inside, and also between the second layer and the medium on the outside. The polymer on heating undergoes changes that absorb heat, release gaseous decomposition products, and alter the thermophysical parameters of the material. When the inside surface of the polymer reaches the failure temperature $T_f$, it starts to move with a speed $v(r)$. The cylinder cools internally and externally when the heating stops (Fig. 1). The processes all repeat during the next heating cycle in accordance with the altered thermophysical characteristics and the temperature distribution.

The following is the heat-balance equation for an elementary volume of the first layer:

$$ q_r c_r \Delta z \Delta r = q_{r+\Delta r} c_{r+\Delta r} (r+\Delta r) \Delta z \Delta r + c_r \Delta r \Delta z \Delta T - H \gamma c_r \Delta r \Delta z - c_g \Delta T \Delta r \sum_{r_k} \frac{\Delta \gamma(r_k, r)}{\Delta T}. $$

(1)

Here the left side is the heat passing through an elementary section at $r$ in time $\Delta \tau$, while the first term on the right is the heat passed by an elementary section at $r + \Delta r$ in time $\Delta \tau$ and the second term is the amount of heat absorbed by the intervening layer in time $\Delta \tau$, which raises the temperature by $\Delta T$, while the third term is the heat absorbed by phase and chemical changes in that volume and the fourth term is the heat needed to raise the temperature by $\Delta T$ in time $\Delta \tau$ for the gases formed in the section $r$ to $R$.

The following assumptions are made: a) the gas escapes from the material almost instantaneously; b) the gas takes up the temperature of the material while passing through the pores; c) the gas does not react with the residue.

Then (1) becomes

$$ \left( c_r - H \frac{\partial \gamma}{\partial T} \right) \frac{\partial T}{\partial \tau} = \lambda \frac{\partial^2 T}{\partial r^2} + \frac{\partial \gamma}{\partial T} \left( \frac{\partial T}{\partial r} \right)^2 + \frac{1}{r} \frac{\partial T}{\partial r} \left( \lambda - c_g \right) \sum_{r_k} \frac{\partial \gamma}{\partial T} \frac{\partial T}{\partial \tau} \frac{\partial T}{\partial \tau} dr.$$

(2)

Here $\lambda$, $c$, and $\gamma$ are dependent on $T$ and on the previous history of the material, while $c_g$ is the specific heat of the gas formed.

<table>
<thead>
<tr>
<th>$T$, K</th>
<th>293</th>
<th>373</th>
<th>473</th>
<th>573</th>
<th>673</th>
<th>773</th>
<th>873</th>
<th>2973</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
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<td>0.84</td>
<td>2.5</td>
<td>4.2</td>
<td>4.2</td>
<td>2.5</td>
<td>0.84</td>
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The initial condition is $T(0, r) = T(r)$.

The boundary conditions are as follows for the internal boundary $\Gamma (r)$ of the material and the external boundary at $r = R$:

$$\frac{\lambda}{\partial T}{\frac{\partial T}{\partial r}} = \alpha_g(T - T_g) + \epsilon_g \left| T^4 - T_g^4 \right|,$$

(3)

$$\frac{\lambda}{\partial T}{\frac{\partial T}{\partial r}} = \lambda_1 \frac{\partial T_1}{\partial r};$$

(4)

$$T(R; \tau) = T_1(R; \tau).$$

Condition (3) neglects reactions between the gas and the residue.

The following are the initial and boundary conditions for the second layer:

$$c_1Y_1 \frac{\partial T_1}{\partial \tau} = \lambda_1 \frac{\partial^2 T_1}{\partial r^2} + \frac{\partial \lambda_1}{\partial T_1} \left( \frac{\partial T_1}{\partial r} \right)^2 + \frac{\lambda_1}{r} \frac{\partial T_1}{\partial r},$$

(5)

$$T_1(0; \tau) = T_1(\tau),$$

(6)

$$T_1(R; \tau) = T(R; \tau),$$

(7)

$$\lambda_1 \frac{\partial T_1}{\partial r} = \frac{\partial T}{\partial r} \text{ for } r = R,$$

(8)

$$\lambda_1 \frac{\partial T_1}{\partial r} = \alpha_v \left( T_v - T_1 \right) + \epsilon_v \left( T_1^4 - T_1^4 \right) \text{ for } r = R_1.$$

Calculations show that the temperature difference across the second layer is only a few degrees for a temperature of several hundred degrees when a thick polymer layer is in contact with a thin highly conducting second layer. We therefore neglect equations (5)-(8) and replace the effect of the second layer on the first by a thermal-capacity term in the boundary condition (4), which becomes

$$\lambda \frac{\partial T}{\partial r} = \alpha_v \left( T_v - T \right) + \epsilon_v \left( T_1^4 - T_v^4 \right) - c_1Y_1 \delta_v \frac{\partial T}{\partial \tau} \text{ for } r = R,$$

(9)

$$\delta_1 = R_1 - R.$$

The equation for the boundary $\Gamma (\tau)$ is

$$\Gamma (\tau) = \frac{r_0}{\delta} \int_{\delta}^{\tau} u(\tau) d\tau,$$

(10)