DETERMINATION OF THE THERMOPHYSICAL CHARACTERISTICS
OF LIQUIDS BY A HEATED-PROBE METHOD

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This is an examination of the heated-probe method used in studying the thermophysical characteristics of viscous liquids over a wide temperature range, under the pressure of saturated vapors in a closed space. The results of the investigation are given.

The determination of thermophysical properties of liquids is associated with a number of difficulties which arise primarily in the formulation of the experiment. In this paper we propose the utilization of probe methods which are widely used [4] in determining the thermophysical characteristics of such materials as liquids, powders, soils under natural conditions, basalt rocks, etc. The probe method is convenient for practical purposes, since it does not require the preparation of complex measuring apparatus, it is very simple to fabricate, and well-developed formulas are available for the determination of the coefficient of thermal conductivity.

We employed the heated-probe method to determine the thermophysical characteristics of liquids over a broad temperature range from 20-180 °C and at the high pressures produced by liquid vapors heated in a closed vessel.

In deriving the calculation formulas for the determination of the coefficients of thermal conductivity, thermal diffusivity, and specific heat capacity we employed a problem [1, 2] which describes the propagation of heat by a linear heat source in a continuous medium.

It is assumed that the cylinder is made from an ideal conductor, that it has a radius r, and that it is surrounded by an unbounded medium; at τ > 0 a quantity of heat equal to q is liberated per unit time per unit length, and the initial temperature of the entire system is equal to zero.

The temperature field of the space is described by the equation

\[ \frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{1}{x} \frac{\partial t}{\partial x} \right), \tag{1} \]

where t is the temperature; \( \tau \) is the time; and x is the radial distance from the linear heat source.

For the solution [1] of Eq. (1) we have

\[ t = \frac{q}{4\pi \lambda} \int_{0}^{\gamma} \left[ \exp \left( -\frac{x^2}{4a\tau} \right) / \tau \right] d\tau. \tag{2} \]

Expression (2) can be written in the form

\[ t = \frac{q}{4\pi \lambda} \int_{0}^{\gamma} e^{-\frac{1}{4Y}} \frac{1}{Y} dY, \tag{3} \]

where the variable \( Y = \alpha \tau/x^2 \).

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Since it is impossible to express the integral in (3) in elementary functions, we will expand $e^{-\sqrt{Y}}$ in series, so that (3) assumes the form

$$
t = \frac{q}{4\pi\lambda} \int_0^\tau \left( \frac{1}{Y} - \frac{1}{Y^2} + \frac{1}{32Y^3} - \cdots \right) dY,
$$

from which

$$
t = \frac{q}{4\pi\lambda} \left( \ln Y + \frac{1}{4Y} - \frac{1}{64Y^2} + \cdots \right).
$$

We see from an analysis of (5) that for large $\tau$ the following approximation is valid:

$$
t = \frac{q}{4\pi\lambda} \ln Y.
$$

We can derive the theoretical formula for the determination of the coefficient of thermal conductivity from (6).

Indeed, for certain rather large time intervals $\tau_1$ and $\tau_2$ satisfying (6), i.e., when the temperature curve is close to the logarithmic and before the steady-state heating regime for the medium has set in, we can write the respective temperature values as follows:

$$
t_1 = \frac{q}{4\pi\lambda} \ln \frac{\alpha\tau_1}{x^2},
$$

$$
t_2 = \frac{q}{4\pi\lambda} \ln \frac{\alpha\tau_2}{x^2}.
$$

Here we have in mind the temperature at the identical point, situated at a radial distance $x$ from the linear heat source.

Then

$$
t_2 - t_1 = \frac{q}{4\pi\lambda} \left( \ln \frac{\alpha\tau_2}{x^2} - \ln \frac{\alpha\tau_1}{x^2} \right)
$$

or

$$
\Delta t = \frac{q \ln \frac{\tau_2}{\tau_1}}{4\pi\lambda},
$$

whence

$$
\frac{q \ln \frac{\tau_2}{\tau_1}}{4\pi\Delta t} = \frac{q}{4\pi\Delta t} \ln \frac{\alpha\tau_2}{\alpha\tau_1}.
$$

Considering the thermal losses and the thermal contact resistance of the probe itself, it is necessary that we introduce into (11) some correction factor which is constant for this probe. Finally, the theoretical formula for the determination of the coefficient of thermal conductivity has the form

$$
\lambda = \beta \frac{q}{4\pi\Delta t} \ln \frac{\tau_2}{\tau_1}.
$$

The probe constant $\beta$ is determined by its preliminary calibration on the basis of standard liquids.

Thus, the determination of the coefficient of thermal conductivity by the heated-probe method involves no particular difficulties and yields completely satisfactory results. However, in actual practice it is necessary to know the frequency, the coefficient of thermal diffusivity and the specific heat capacity of the material. In this paper we propose an experimental formula for the determination of the coefficient of thermal diffusivity.

As a result of numerous experiments we have noted that the temperature recorded at the linear heat source for the liquid being investigated is inversely proportional to the thermal diffusivity of that liquid. Consequently, for the exact same instant of time we can write

$$
\frac{a_1}{a_2} = \frac{t_2}{t_1},
$$

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