The solution to the Boltzman equation is obtained by the Chapman–Enskog method for the case of low temperatures. An expression is found with the help of the quantum "diffusion" model for the thermal conductivity of a gas mixture at low temperatures.

Molecular translational degrees of freedom are "frozen in" at low temperatures, at a time when the molecular spin makes some contribution to the thermal conductivity of the mixture, i.e., the degree of freedom associated with the internal motion.

Both monatomic and polyatomic molecules at low temperatures therefore exhibit internal degrees of freedom. The translational degrees of freedom are "unfrozen" with a rise in temperature and beginning at about 200°C, we must begin to give consideration to the contribution of the translational degrees of freedom to the heat conduction of the mixture.

The conduction of heat in a gas mixture at low temperatures is brought about by quantum "diffusion" [5].

In calculating the low-temperature heat conduction of a mixture, we must bear in mind the quantum effect. Quantum theory introduces two variations into the classical kinetic theory of gases.

1. Collisions between two gas molecules must be treated from the standpoint of the quantum theory of collisions. The deflection of the relative-velocity vector as a consequence of the collisions of a pair of molecules with masses \( m_1 \) and \( m_2 \) is approximately the same as in the assumption – resorting to classical theory – that each molecule is surrounded by a "wave" field whose linear extent is on the order of the "wavelength" [1, 2]

\[
\tilde{\lambda} = \frac{h}{2\pi \sqrt{\mu kT}}.
\]

This quantity is the greater, the lighter the molecule and the lower the temperature. The quantity \( \tilde{\lambda} \) is known as the de Broglie wavelength and characterizes deviation from the classical theory for the case in which it is commensurate with or greater than the molecule diameter.

2. The second variation of classical theory is associated with the change in the Maxwell–Boltzmann equilibrium distribution function for the Bose–Einstein and Fermi–Dirac laws.

At very low temperatures, the wave fields associated with the molecules become very much larger than the molecules and it is possible to have a state in which the lowest quantum levels are occupied. A gas in this state of level-occupation is referred to as degenerate.

The first variation (the diffraction effect) is significant when the de Broglie wavelength is on the order of the molecular dimensions.

The second variation (the symmetry effect) is significant when the de Broglie wavelength is on the order of the distance between the molecules in the gas.

Unlike the first quantum effect, the second effect will be smaller at ordinary temperature and becomes important only at very low temperatures (below 2K) and at high densities.
The state of the \( \nu \)-component gas mixture is completely described by the distribution function \( f(r, v, t) \), which is a solution of the integrodifferential Boltzmann equation which, with consideration of the quantum effects of the gas mixtures whose molecules exhibit internal degrees of freedom, has the form

\[
\frac{\partial f_{q_{ij}}}{\partial t} + v_q \frac{\partial f_{q_{ij}}}{\partial r} = \sum_q \sum_{jkl} \iint f_{q_{ij}} f_{q_{ij}} (1 + \theta_{qf_{q_{ij}}})(1 + \theta_{qf_{q_{ij}}})
- f_{q_{ij}} f_{q_{ij}} (1 + \theta_{qf_{q_{ij}}})(1 + \theta_{qf_{q_{ij}}}) g_{q_{ij}} \sin \theta \ d\lambda dq dv_{q_{ij}}.
\]

(1)

Here \( \theta = \hbar/mq_i^3(\delta/Gq_i) \), \( \delta = -1, 0, \) and \( +1 \), respectively, for the statistics of Fermi–Dirac, Maxwell–Boltzmann, and Bose–Einstein; \( g_{q_{ij}} \) is the differential effective scattering cross section for the molecules and the subscripts \( i \) and \( j \) denote the molecular state prior to collision, with the superscripts \( k \) and \( l \) denoting the molecular state subsequent to the collisions.

The equilibrium distribution function is given by (1) in which the right-hand member is equal to zero. For quantum systems we have various equilibrium distribution functions:

for the Maxwell–Boltzmann statistic

\[
f_{q_{ij}}^{(0)} = A_q \exp \left[ - \frac{m_q V_q^2}{2kT} + \epsilon_{q_{ij}} \right],
\]

for the Bose–Einstein statistic

\[
f_{q_{ij}}^{(0)} = \left( A_q \exp \left[ \frac{m_q V_q^2}{2kT} + \epsilon_{q_{ij}} \right] - \theta_q \right)^{-1},
\]

for the Fermi–Dirac statistic

\[
f_{q_{ij}}^{(0)} = \left( A_q \exp \left[ \frac{m_q V_q^2}{2kT} + \epsilon_{q_{ij}} \right] + \theta_q \right)^{-1}.
\]

Let

\[
f_{q_{ij}}^{(0)} = \left( A_q \exp \left[ \frac{m_q V_q^2}{2kT} + \epsilon_{q_{ij}} \right] \right)^{1},
\]

where

\[\epsilon_{q_{ij}} = E_{q_{ij}}/kT.\]

We will seek the approximate solution of the Boltzmann equation in the form

\[f_{q_{ij}} = f_{q_{ij}}^{(0)}(1 + \phi_{q_{ij}}),\]

(3)

where \( \phi_{q_{ij}} \) is the function of the velocity, the temperature, and the pressure gradients.

Relationship (3) is substituted into (1) and the terms of second order are neglected \([1, 3, 6]\). Having introduced the substitution \( \beta_{q_{ij}} = \phi_{q_{ij}}/(1 + \theta_{qf_{q_{ij}}}) \) \([3]\), we obtain

\[
\frac{\partial f_{q_{ij}}^{(0)}}{\partial t} + v_q \frac{\partial f_{q_{ij}}^{(0)}}{\partial r} = \sum_q \sum_{jkl} \int \int \int f_{q_{ij}}^{(0)} f_{q_{ij}}^{(0)} (1 + \theta_{qf_{q_{ij}}}) (1
+ \theta_{qf_{q_{ij}}})(1 + \theta_{qf_{q_{ij}}}) g_{q_{ij}} \sin \theta \ d\lambda dq dv_{q_{ij}}.
\]

(4)

The conditions for the solution of (4) have the form

\[
\sum_q \int f_{q_{ij}}^{(0)} (1 + \theta_{qf_{q_{ij}}}) \beta_{q_{ij}} dv_{q_{ij}} = 0,
\]

\[
\frac{1}{\rho} \sum_q \sum_{q_{ij}} m_q \int f_{q_{ij}}^{(0)} (1 + \theta_{qf_{q_{ij}}}) \beta_{q_{ij}} v_q dv_{q_{ij}} = 0.
\]

(5)

\[
\frac{1}{\rho} \sum_q \sum_{q_{ij}} \int f_{q_{ij}}^{(0)} (1 + \theta_{qf_{q_{ij}}}) \left[ \frac{1}{2} m_q (v_q - v_0)^2 + E_{q_{ij}} \right] \beta_{q_{ij}} dv_{q_{ij}} = 0.
\]