THE THERMAL CONDUCTIVITY OF GRANULAR AND
WEAKLY SINTERED MATERIALS

G. N. Dul'nev, Yu. P. Zarichnyak,
and B. L. Muratova

We propose a new stable model of a granular or sintered material in a form with a second-
order structure, with mutually penetrating components. We derive the functional relation-
ships for the calculation of the coefficients of generalized conductivity* for the granular and
sintered materials.

1. Discussion of the Need for the Construction of a
New Model of Granular Systems

Numerous books, surveyed in [1-6], have been devoted to a study of the process of heat transfer in
granular systems. The adequacy of the physical model and of the real system are usually evaluated on the
basis of the extent to which the calculation results coincide with the experimental data. However, agree-
ment with experiment, despite its value, is by no means a definitive criterion for the quality of a given
model or calculation scheme, since the comparison is not always sufficiently extensive to cover the entire
possible range of variation in the significant parameters. The testing of any given formula by other re-
searchers with different materials in another range of variation for the determining parameters, on
occasion, reveals substantial differences between the calculation results and those of the experiment.

Satisfactory agreement in limited comparison with experiment may be a random occurrence, or it
may be explained by the mutually offsetting effect of individual defects within the model, or it may be a
consequence of an inaccuracy in the mathematical conversions. In addition to comparing the calculation
results with the experiment—an absolutely mandatory requirement—we should devote some attention to
the internal adequacy of the model. With regard to granular systems, the internal adequacy of the model ap-
parently should be understood to refer to the extent to which the model corresponds to the real structure
of the system, its physical stability, and its isotropy.

One method of determining the internal adequacy of a model is the testing of the working formula
derived from that model on the limit transitions. In the limiting cases, with a porosity of \( m_p \approx 0.26 \) (models
with a tetrahedral packing) and \( m_p \approx 0.47 \) (models with cubic packing), many of the working formulas lead
to physically absurd results. Thus, for example, the functional relationships for the calculations of thermal
conductivity \([7, 8]\) for \( m_p = 0.26 \) yield a value of \( \lambda = \infty \) for the effective thermal conductivity; in \([4, 7, 9-12]\),
as \( m_S \rightarrow 1 \), we are told that \( \lambda < 0 \) or \( \lambda = 0 \), or we are given a positive but physically invalid result. For
the case in which \( \lambda_D/\lambda_S = 1 \) we should expect \( \lambda = \lambda_S = \lambda_D \). However, the working formulas from \([4, 7, 9, 11-13]\)
do not yield the anticipated result. All of the formulas derived from models of noncontacting in-
clusions of any shape, distributed through the matrix, with \( \lambda_D = 0 \) yield \( \lambda = 0 \), which contradicts the experi-
mental data. Let us draw attention to another two important factors—the isotropicity of the model and its
physical stability, factors virtually not touched upon in the above-cited surveys \([1-6]\). The random nature
of the structure in real granular systems determines the isotropicity of their properties. This fact must
be borne in mind in devising ordered models of a granular system, in choosing the form of its symmetry,
and in selecting the "elementary cell," for which we undertake the mathematical description of the process
within the volume of that cell. Isotropicity for the properties of the granular system can be achieved in

*The coefficient of generalized conductivity will subsequently be understood to refer to any of the coeffi-
cients of thermal or electrical conductivity, to the permittivity or to magnetic permeability.
two ways: a) by the construction of an isotropic elementary cell; b) by the use of an anisotropic elementary cell, with subsequent analytical averaging of its properties in all directions.

2. The Stability of Models of Granular Systems

The most frequently used models of granular systems in the form of spherical particles with any ordered packing are strictly stable only for a single specific value of the component volume concentration (for example, \( m_p \approx 0.26 \) for tetrahedral packing, \( m_p \approx 0.47 \) for cubic packing, etc.). It is impossible to substantiate the stability of models of granular systems in the form of noncontacting inclusions of any shape (the solid component), distributed within the gas component [3, 7, 8, 10-13]. These models are suitable, more likely, for the description of aerosol structure (smokes and fogs).

We can cite the following reasons which, in our opinion, partially explain the stability of real mono-disperse granular systems, when their porosity differs from the theoretical values: the combined packing (individual segments of the system are formed by various types of packing); deviations in the shape of real particles from that of the simplest geometric figures and the existence of microirregularities; the existence of a more complex system structure formed by segments with a dense grain packing and a space lattice of larger voids, penetrating the entire volume of the granular system.

It is natural to assume that in real granular materials all of the above-enumerated three factors apply.

1) The presence of combined packing (a combination of tetrahedral and cubic packing, or others) provides a substantiation of the stability of a model with spherical grains only for a narrow range of variation in porosity from \( m_p \approx 0.26 \) to \( m_p \approx 0.47 \). With a porosity of \( m_p > 0.47 \) no manner of ordered packing for the spherical particles in contact results in a physically stable model and the separate grains seemingly float in space.

2) Measurements of porosity for real granular materials with grains of various configurations (spheres, cylinders, cubes, ellipsoids of revolution, etc.) demonstrated that the porosity of such systems depends weakly on particle shape, and the microirregularities have virtually no effect on the porosity of the granular system [1, 7]. Consequently, deviations in grain shape from the spherical and the existence of microirregularities cannot serve as a basis for the stability of granular systems in the case of high porosity.

3) Direct observation of various granular systems by means of a binocular microscope makes possible schematic two-dimensional representation of structure (see Fig. 1a). It can be seen that the granular system is formed by a "skeleton," consisting of a disordered but relatively dense grain packing (a first-order structure) and larger voids, penetrating the skeleton and forming — together with the skeleton — a