The Basis of the Dependence of Burning Rate on Pressure in the Relaxation Theory of Propagation of Combustion

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It is shown that the physical basis of the dependence of burning rate on pressure, lacking in the relaxation theory of propagation of combustion, actually forms part of the relaxation mechanism itself. The corresponding formulas of the relaxation theory are refined.

It is noted in [2] that the cause of the influence exerted by pressure on burning rate should be sought chiefly in the pressure dependence of the temperature $T_g$ at the interface between the condensed and gas phases, but this dependence was not given a physical basis.

In [1] (Fig. 3) the thermal conditions that must prevail in the condensed phase during the combustion process were examined. We may supplement the remarks in [1] by noting that to realize the relaxation mechanism of propagation of combustion, it is not necessary to have ignition; it is sufficient to have rapid exothermic expansion of the component, designated by circles the figure, when a certain temperature is reached. This temperature will be equivalent to the ignition point. Exothermic reduction of this component is also a possibility.

Since ignition of the next elementary layer occurs within the condensed phase, high-temperature gases are formed in the surface layer of the substance and create a rapidly rising pressure. On reaching a certain pressure, they force open the surface layer, suddenly expand to the pressure of the external medium, whereupon their temperature drops to $T_g$. The higher the external pressure, the less the degree of expansion of the gases, and the higher the temperature $T_g$, i.e., $T_g$ must depend on the pressure. Thus, the physical basis for the pressure dependence of the burning rate forms part of the relaxation mechanism itself and does not require additional assumptions.

Obviously, when the gases force open the surface layer, they must reduce the latter to fine particles and eject them into the gas phase, forming a gas-smoke mixture. Therefore, a necessary feature of the relaxation mechanism of combustion propagation must be a smoke-gas mixture in the gaseous phase. All the components of the initial materials may occur as solid particles in the smoke-gas mixture, including partially unburnt particles of fuel and some products of the primary reactions. On the surface of the condensed phase there must form a region of primary reaction products with a relatively low temperature $T_q$, in which there is still little or no heat release ("dark zone"), while beyond the limits of this zone secondary, essentially exothermic reactions must take place in the smoke-gas mixture — further combustion of fuel particles and products of incomplete oxidation. The thermal effect of the secondary reactions may be much greater than that of the primary reactions, since only a small part of the fuel should be consumed before the disruption of the surface layer. The lower the pressure, the longer the "dark zone," not only because of the influence of pressure on the rate of the chemical reactions, but also because the smoke-gas mixture must be ejected to a greater distance [3-5].

Disruption of the surface layer must occur when the difference between the pressure within the material and the external pressure reaches a certain level. This pressure difference will depend on the strength of the disrupted layer, but not on the external pressure, since there is no reason why it should. Therefore,

$$p_q - p_g = p_p = \text{const.} \quad (1)$$

The temperature $T_q$ of the primary reaction products up to disruption of the material is determined by the thermal effect of these reactions and by the specific heat of their products (if we neglect heat transfer during their formation, which is valid, as a first approximation, because ignition time is so short) and should not depend on the pressure $p_q$ i.e., we may assume that $T_q = \text{const.}$

Assuming that the primary reaction products are a perfect gas and that their expansions are adiabatic, we may consider that the temperatures $T_g$ and $T_q$ are related by the Poisson equation. In that case

$$T_g = T_q \left( \frac{p_g}{p_q} \right)^\nu = T_q \left[ \frac{p_q}{(p_q + p_p)} \right]^\nu, \quad (2)$$

where $\nu = (C_p - C_v)/C_p$. For monatomic gases $\nu = 0.38$, for diatomic $\nu = 0.29$, and for triatomic gases and beyond $\nu = 0.25$. All values of $\nu$ between 0.25 and 0.38 are possible, since the primary reaction products may contain gases of different atomic weights in various proportions.
Assuming two different pressures $p_g$ and corresponding temperatures with subscripts 0 and 1, we obtain

$$T_{g1} = T_{g0} \left( \frac{p_g}{p_{g0}} \right)^{\gamma} \left( \frac{p_0 + \Delta p_0}{p_0 + \Delta p_0} \right)^{\gamma} \left( \frac{p_1 + \Delta p_1 + p_p}{p_1 + \Delta p_1 + p_p} \right)^{\gamma},$$

(3)

if temperature is measured in °K.

If the exhaust gases meet appreciable resistance in the exhaust channel, then the pressure $p_g$ will be greater than the external pressure $p$ by the amount of the pressure drop in the exhaust channel $\Delta p$, i.e.,

$$p_g = p + \Delta p.$$

(4)

From (3), taking account of (4) and the equation for the rate of propagation of combustion [1], we obtain

$$\frac{u_1}{u_0} = C \left[ A \left( p_1 + \Delta p_1 \right)^{\gamma} \left( p_0 + \Delta p_0 \right)^{\gamma} - 1 \right] + B,$$

(5)

$$C = \frac{T_{e0} - T_1 + \gamma c + 4a/c \gamma d}{T_{e1} - T_1 + \gamma c + 4a/c \gamma d}, \quad A = \frac{T_{g0} + 273}{T_{e0} - 0.4T_{e0} - 0.6T_1},$$

$$B = \frac{T_{g0} - 0.4T_{e1} - 0.6T_1}{T_{g0} - 0.4T_{e0} - 0.6T_1}.$$

If the ignition temperature depends only slightly on pressure, which should be true in most cases, or $(\gamma c + 4a/c \gamma d) \gg (T_e - T_i)$, then the values of $B$ and $C$ must be close to unity. Moreover, in many cases it may be assumed that $\Delta p_0$ and $\Delta p_1$ are negligibly small compared with $p_g$ and $p_1$. Under these conditions

$$\frac{u_1}{u_0} = A \left( \frac{p_1}{p_0} \right)^{\gamma} \left( \frac{p_0 + p_p}{p_1 + p_p} \right)^{\gamma} - 1 + 1,$$

(6)

When $p_p = 0$, Eq. (5) gives $u_1/u_0 = BC$, while Eq. (6) gives $u_1/u_0 = 1$, i.e., in the latter case the burning rate does not depend on pressure, while in the former case it does, but only very slightly.

When $p_p \to \infty$, Eqs. (5) and (6) coincide with Eqs. (1) and (2) of [2]. For practical calculations Eq. (6) is quite satisfactory.

All possible values of the function $u_1/u_0 = f(p_1/p_0)$ with $p_p$ varying from zero to infinity are included with the limits defined by the above formulas. The upper limit cannot be reached, since at $p_p = \infty$ disruption of the surface layer is impossible, which means that combustion is also impossible. The lower limit at $p_p = 0$ can be reached in theory, and evidently in practice.

The value of $p_p$ can be roughly estimated from the formula

$$p_p \approx \sigma_p \frac{S_1}{S_2}.$$

(7)

Apparently, for the materials in question the ultimate strength may not lie outside the range $5 \leq \sigma_p \leq 49 \cdot 10^6$ N/m². Usually the volume of fuel particles in 4-12% of that of the material. For 4%, $S_1/S_2 \approx 9$, and for 12%, $S_1/S_2 \approx 4$. Therefore the pressure $p_p$ may lie in the range $19.6 \cdot 10^6$ to $44 \cdot 10^6$ N/m², but in practice it must lie within much narrower limits. The figure shows curves computed from Eq. (6). It is clear that if $p_p$ is varied by a factor of five, then at $p_1/p_0 = 4000$, $u_1/u_0$ varies from 10 to 17, i.e., by a factor of only 1.7. At smaller values of $p_1/p_0$, and also when $v < 0.3$ and $A < 3$, the variation of $u_1/u_0$ will be less. Thus, an accurate determination of $p_p$ is not necessary; an approximate value will give only a small error.

The actual combustion process differs from that assumed in deriving the equations, and one should know to what extent this difference affects the formulas obtained.

In the first place, the process of expansion of the gases is not adiabatic, since, during the ignition time, part of the energy liberated must go toward heating the condensed phase and disruption of the surface layer. It is to be expected,