Calculation of Radiative Heat Transfer with Anisotropic Dispersion of Radiant Energy

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Opticogeometrical resolvents and their equations are established for the case of anisotropy in the optical properties of a medium and its surfaces.

It is well known that the optical properties of all materials depend very strongly on the direction of incident and transmitted rays, the latter having an appreciable effect on the resultant radiant fluxes [1]. The advantages of thermal radiation cannot always be fully utilized in modern technological processes, meanwhile, because no complete general theory has been developed yet on calculating the radiative heat transfer where the optical properties of solid surfaces and semitranslucent media are anisotropic [1-8]. This study here is concerned with an analysis of the problem.

It is assumed here that there occurs no spectral redistribution of radiant energy due to dispersion. In the case of a closed system which consists of anisotropically radiating and dispersing boundary surfaces and media contained between them, we obtain the following integral equations for the luminance of effective radiation from elements of boundary surfaces and of the media contained between them:

\[ B_e \left( \frac{M}{r_0} \right) = B_e \left( \frac{r}{r_0} \right) + \int B_e \left( \frac{c_1}{r (c_1, M)} \right) R \left( \frac{M}{r (c_1, M)} \right) \gamma \times \left( \frac{M}{r (c_1, M)} \right) K_1 (c_1, M) \exp \left( \int_{r (c_1)} -k (r) \, dr \right) \, dF (c_1) \]

\[ + \int B_e \left( \frac{n_1}{r (n_1, M)} \right) R \left( \frac{M}{r (n_1, M)} \right) \gamma \left( \frac{M}{r (n_1, M)} \right) K_2 (n_1, M) \exp \left( \int_{r (n_1)} -k (r) \, dr \right) \, dV (n_1), \]

\[ B_e \left( \frac{b}{r_0} \right) = B_e \left( \frac{b}{r_0} \right) + \int B_e \left( \frac{c_1}{r (c_1, b)} \right) \beta \left( \frac{b}{r (c_1, b)} \right) \gamma \times \left( \frac{b}{r (c_1, b)} \right) K_3 (c_1, b) \exp \left( \int_{r (c_1)} -k (r) \, dr \right) \, dF (c_1) \]

\[ + \int B_e \left( \frac{b}{r (n_1, b)} \right) \beta \left( \frac{b}{r (n_1, b)} \right) \gamma \left( \frac{b}{r (n_1, b)} \right) K_4 (n_1, b) \exp \left( \int_{r (n_1)} -k (r) \, dr \right) \, dV (n_1), \]

where

\[ K_1 (c_1, M) = \cos \left( \frac{\hat{r} (c_1, M) \cdot \hat{n} (M)}{\left| \pi r^2 (c_1, M) \right|^2} \right), \]

\[ K_2 (n_1, M) = \cos \left( \frac{\hat{r} (n_1, M) \cdot \hat{n} (M)}{\left| \pi r^2 (c_1, M) \right|^2} \right). \]
\[ K_3 (c,b) = \cos \left( \frac{n_1 (c_1)}{r (c_1 b)} \right) \left( \frac{4\pi^2 (c,b)}{r (c_1 b)^2} \right)^{-1} \]

\[ K_4 (n,b) = \left( \frac{4\pi^2 (n,b)}{r (n,b)^2} \right)^{-1} \]

The solution of Eqs. (1), (2) yields the following expression for the luminance of effective radiation along any direction \( r_0 \) from elements of boundary surfaces and the media contained between them:

\[
B_a \left( \frac{M}{r_0} \right) = B_e \left( \frac{M}{r_0} \right) + \int_{V_1} B_e \left( \frac{c_1}{r (c_1 b)} \right) R \left( \frac{M}{r (c_2 M)} \right) \gamma \\
\times \left( \frac{M}{r (c_2 M) r_0} \right) K_1 (c_1, M) \exp \left( \int_{r(c_1)}^{r(c_2)} -k (r) \, dr \right) dF (c_1) \\
+ \int_{V_1} \int_{V_2} B_e \left( \frac{c_1}{r (c_1 c_2)} \right) \varphi_{11} \left( \frac{c_1 c_2 M}{r (c_1 c_2) r (c_2 M) r_0} \right) dF (c_1) dF (c_2) \\
+ \int_{V_1} \int_{V_2} B_e \left( \frac{c_1}{r (c_1 n_2)} \right) \varphi_{22} \left( \frac{c_1 n_2 M}{r (c_1 n_2) r (n_2 M) r_0} \right) dF (c_1) dV (n_2)
\]

\[
+ \int_{V_1} \int_{V_2} B_e \left( \frac{n_1}{r (n_1 M)} \right) R \left( \frac{M}{r (c_1 M)} \right) \gamma \left( \frac{M}{r (c_1 M) r_0} \right) K_3 (n_1 M) \exp \left( \int_{r(c_1)}^{r(n_1)} -k (r) \, dr \right) dV (n_1)
\]

\[
+ \int_{V_1} \int_{V_2} B_e \left( \frac{n_1}{r (n_1 c_2)} \right) \varphi_{31} \left( \frac{n_1 c_2 M}{r (n_1 c_2) r (c_2 M) r_0} \right) dV (n_1) dF (c_2)
\]

\[
+ \int_{V_1} \int_{V_2} B_e \left( \frac{n_1}{r (n_1 n_2)} \right) \varphi_{41} \left( \frac{n_1 n_2 M}{r (n_1 n_2) r (n_2 M) r_0} \right) dV (n_1) dV (n_2)
\]

The opticogeometrical functions \( \varphi_{ik} \) represent the effect of innumerable dispersions of radiant energy at the boundary surfaces and inside the volume of the media in a system, and they are determined from the following system of opticogeometrical integral equations:

a. for boundary points on the surfaces

\[
\varphi_{11} \left( \frac{c_1 c_2 M}{r (c_1 c_2) r (c_2 M) r_0} \right) = K_{11} \left( \frac{c_1 c_2 M}{r (c_1 c_2) r (c_2 M) r_0} \right)
\]

\[
+ \int_{V_1} K_1 (c_2 c_2) \varphi_{11} \left( \frac{c_1 c_2 M}{r (c_1 c_2) r (c_2 M) r_0} \right) dF (c_2)
\]