MASS AND HEAT TRANSFER IN BUILDING MATERIALS

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A system of differential equations is proposed for mass and heat transfer in capillary-porous bodies. The principal thermophysical characteristics of mass and heat exchange in relation to building materials and structures are discussed.

Building materials are moist capillary-porous bodies, in which heat transfer is inseparably associated with mass transfer of moisture (vapor and liquid) and air. Therefore heat transfer calculations relating to building materials must take mass transfer into account. These calculations call for extensive use of machine computation and the application of generalized variable methods (similarity theory). It is therefore necessary to have a sufficiently rigorous mathematical formulation of the problem and a clear understanding of the corresponding thermophysical characteristics.

Physical characteristics of the system. Consider a capillary-porous body with a lyophilic skeleton, i.e., one in which the walls of the capillaries and pores sorb gas, vapor and liquid; an osmotic-diffusion bond may then be formed between the substance sorbed and the skeleton.

A system of this kind differs substantially from the dispersed media models examined in classical filtration theory. The substance bound by the capillary-porous skeleton may be in the form of a liquid, vapor, inert gas, solid, or supercooled liquid, depending on the conditions of mass and heat transfer. The freezing point of a liquid varies over a wide range, depending on the type of bond between substance and skeleton. There is therefore always a certain amount of supercooled liquid (water) in capillary-porous bodies at temperatures below 0°C.

It should also be noted that in most cases the pores and capillaries are not completely full of liquid or ice, but partly filled with a vapor-gas mixture. To simplify the analysis we shall assume the liquid to be water without any dissolved substance. The presence of dissolved substances modifies the mass transfer processes and causes a number of other effects.

The main parameter of a capillary-porous material is its moisture content $u$, or the sum of the relative concentrations of the $i$-th bound substance ($i = 1, 2, 3, 4$),

$$u = \sum_{i} u_i = \frac{\omega}{\gamma} = \frac{1}{\gamma} \sum_{i} \omega_i.$$  \hspace{1cm} (1)

If shrinkage of the body is neglected, the volume concentration of the skeleton will equal the density of the absolutely dry material ($\gamma = \gamma_0$). If the skeleton walls are slightly hydrophilic, and changes in the density of the liquid of the monomolecular adsorbed layer are neglected, then the amount of moisture in the physico-mechanical bond (capillary moisture, ice, vapor, and air) may be determined from the relation

$$\omega_i' = \Pi_{i} b_i \rho_i = \gamma_0 u_i',$$  \hspace{1cm} (2)

where $b_i$ is the degree of filling of pores and capillaries, or the ratio of the volume of the $i$-th bound substance to the pore volume*:

$$\Pi_{i} = \int_{r_{s}}^{r_{\text{max}}} f_{i}(\rho) \, dr;$$

is the volume porosity of the body; $f_{i}(\rho)$ is the differential pore distribution curve. The relation $\sum b_i = 1$ holds for $b_i$. It should be noted that (2) will not be valid for the moisture of the physico-chemical bond ($\omega_i = \omega_i'; u_i = u_i'$). The equality $\omega_i = \omega_i'$, $u_i = u_i'$, and hence (2), can be assumed only for typical capillary-porous materials with a small hygroscopic moisture content.

For the molar (hydrodynamic) motion of moisture (liquid, gas or vapor) through the pores that occurs in filtration processes the flow density of the $i$-th substance is

$$j_{i, \text{mol}} = \Pi_{i} \rho_i b_i w_i;$$  \hspace{1cm} (3)

*In filtration theory $b_i$ is called the saturation.
where $\Pi S$ is the surface porosity of the body, or the ratio of the area of all the pores (holes) to the area of the cross section in a direction perpendicular to the vector $j_1$ mol.

For a polycapillary-porous body $\Pi S = \int_0^{\max} f_s(r) \, dr$, where $f_s(r)$ is the differential distribution curve of the surface porosity of the body with respect to the radius of the pores. For homogeneous bodies the surface porosity $\Pi S$ is approximately equal to the volume porosity $\Pi V (\Pi V = \Pi S = \Pi)$.

Strictly speaking, the $b_i$ in (3) is different from the $b_i$ in (2). However, when $\Pi V = \Pi S = \Pi$, $b_i$ will denote the degree of filling of the pores of the $i$-th bound substance. Thus we may write

$$j_{i \text{ mol}} = \Pi_i b_i w_i.$$  

Relation (4) may be used to determine the flow density of the capillary moisture in a monocapillary-porous body. In that case $w_i = w_{\text{cap}}$, where $w_{\text{cap}}$ is the linear velocity of the liquid in a monocapillary-porous material under the action of the capillary forces.

The following circumstance is also important. Under ordinary conditions, when the pressure of the moist air in the pores of the material is only slightly different from barometric, the mass of the air and vapor in the pores is negligibly small compared to that of the liquid or ice. Naturally, this presupposes that the material is in equilibrium with the surrounding moist air, i.e., its moisture content differs from zero. According to Poznots's calculations, under normal conditions for materials of maximum porosity (brick, wood, etc.) the mass of moist air in the pores of the material is about $10^{-5}$% of the mass of liquid corresponding to the equilibrium moisture content. Therefore the total moisture content of the material $u$ may be regarded as equal to the moisture content of liquid $u_2$ and ice $u_3$:

$$u = \sum_i u_i = u_2 + u_3.$$  

(5)

We shall use this relation in calculating sources of bound matter due to phase transitions.

Differential equations of mass and heat transfer. From the law of conservation of mass of bound matter we obtain the differential equation of mass transfer

$$\frac{\partial (u_i \Pi)}{\partial \tau} = - \text{div} (j_{\text{diff}} + j_{\text{mol}}) + I_i,$$  

(6)

where the relation $\sum_i I_i = 0$ holds for $I_i$.

Neglecting shrinkage ($\gamma = \gamma_0 = \text{const}$) and making use of (4), we get:

$$\gamma_0 \frac{\partial u_i}{\partial \tau} = - \text{div} (j_{\text{diff}} + \Pi_i b_i w_i) + I_i.$$  

(7)

We obtain the differential equation of heat transfer from the equation of internal energy transfer

$$c \gamma_0 \frac{\partial T}{\partial \tau} = - \text{div} j_q + \sum_i h_i I_i - \sum_i (j_{\text{diff}} + \Pi_i b_i w_i) c_i \nabla T,$$  

(8)

where $c$ is the reduced specific heat of the body (i.e., $c = c_0 + \sum_i c_i u_i$); $h_i$ is the specific enthalpy of the $i$-th bound substance ($h_i = \Delta h_i / \Delta T$); $j_q$ is the heat flux density due to Fourier-law thermal conduction ($j_q = -\lambda \nabla T$).

Equations (7) and (8) form the most general system of differential equations of mass and heat transfer in capillary-porous bodies, from which the equations for a number of special cases may be obtained: the filtration equation, the equations of moisture and heat transfer in monocapillary-porous materials, and so on. Vapor diffusion in capillary-porous bodies is determined by the moisture content and temperature gradients:

$$j_{\text{diff}} = a_{m1} \gamma_0 u + a_{m2} \gamma_0 \nabla T.$$  

(9)

The flow of capillary moisture is proportional to the gradient of capillary potential, which in turn can be expressed in terms of the moisture content and temperature gradients, i.e., the motion of capillary moisture can be considered as capillary diffusion.

*For a solid the specific heat at constant volume $c_V$ is usually assumed to be equal to the specific heat at constant pressure, $c_p$, i.e., $c_p = c_V = c$. 

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