HEAT TRANSMISSION THROUGH OUTSIDE WALLS

I. P. Zhuk and L. I. Pisarchuk
Inzhenerno-Fizicheski Zhurnal, Vol. 8, No. 2, pp. 170-181, 1965

The problem of heat transmission in multi-layer walls is analyzed. It is shown that the officially recom-
mended method requires improvement and new design formulas are proposed.

Building materials are capillary-porous bodies containing moisture in the form of liquid, vapor, and, at sub-zero
temperatures, ice. It has been shown [1, 5] that the problem of heat transmission in buildings is closely related to ques-
tions of heat and mass transfer. Therefore, to find the heat transmission through outside walls, it is necessary to solve the
complete heat and mass transfer equations:

\[ \frac{\partial \gamma u_i}{\partial \tau} = \text{div} j_i + I_i, \]  
\[ c_i \frac{\partial t}{\partial \tau} = \text{div} (\lambda \Delta t) + r_i l_i. \]  

Here we shall neglect heat transfer due to filtration of vapor-gas mixture. It has been shown [4] that this assumption is
justified for \( K_f < 0.5 \). In the positive temperature range \( t > 0^\circ C \) the liquid source \( I_2 \) is conditioned by condensa-
tion of vapor. According to Posnov's calculations, under normal conditions the mass of moist air in the capillaries is 10^{-5}\% of
the mass of liquid corresponding to equilibrium moisture content. Then it is accurate enough to make the specific
mass content equal to the specific mass content of liquid \( u = u_0 \). From Eq. (1) we obtain

\[ I_1 = \text{div} j_i = \frac{\partial}{\partial \tau} \left( \frac{\epsilon D \rho \frac{M_1}{M} \nabla q}{\rho M} \right), \]  

where \( \epsilon \) is the resistance coefficient for internal diffusion of vapor, and \( D \) the diffusion coefficient of vapor in air. For a
building material in the moist state, the partial pressure of vapor \( p_s \) in the capillaries is equal to the saturation pressure
and is a function of temperature. Then

\[ I_1 = -\epsilon D \frac{M}{\rho M} \frac{\partial p_s}{\partial T} \nabla^2 t. \]  

Assuming that the thermophysical characteristics depend very little on the coordinates, we may write (2) in the form

\[ \frac{\partial t}{\partial \tau} = a_{eq} \frac{\partial^2 t}{\partial x^2}, \]  

where \( a_{eq} \) is the equivalent thermal diffusivity:

\[ a_{eq} = \left( \frac{\lambda + \epsilon D_{12} \frac{RT}{M_1} \frac{\partial p_s}{\partial T}}{c_i} \right). \]  

It is shown in [4, 5] that (5) may be used to calculate the heat transmission through outside walls, when \( a_{eq} \), cal-
culated from (6), is taken as \( a \). In the presence of a considerable moisture gradient over the thickness of the wall, it should
be divided into layers, and the mean characteristics of each layer used.

We shall employ (5) to analyze heat transmission through a multi-layer wall. Heat transmission through an outside
wall is usually unsteady. In the first approximation we shall assume that the temperature of the outside air (outside sur-
facer) varies according to a harmonic law.

Mathematically, the problem of an \( n \)-layer wall is described by the following system of equations:

\[ \frac{\partial t_i}{\partial \tau} = a_{eq} \frac{\partial^2 t_i}{\partial x^2}, \quad 0 < x < l_i; \]  
\[ \ldots \ldots \ldots \ldots \ldots \ldots \]  
\[ \frac{\partial t_n}{\partial \tau} = a_{eq} \frac{\partial^2 t_n}{\partial x^2}, \quad l_{n-1} < x < l_n; \]  

\[ 110 \]
\[
\begin{align*}
t_i(\tau, l_i) &= t_{i+1}(\tau, l_i); \\
\frac{l_i}{\lambda_{i+1}} \frac{\partial t_i}{\partial x} &= \frac{\partial t_{i+1}}{\partial x} \bigg|_{x=l_i} \quad (i = 1, 2, ..., n-1)
\end{align*}
\]

with appropriate boundary and initial conditions. In [6, 7] Eqs. (7) have been solved for a two-layer wall, and in [8] for the n-layer case.

In this paper we shall examine Eqs. (11) and (12) from [6], and (5), (6), and (7) from [8].

We shall write expressions for the temperatures and heat fluxes in two-layer and three-layer systems.

**Two-layer wall.** An outside layer of thickness \( l_1 \) is in perfect thermal contact with a second layer of considerably greater thickness (\( l_2 \gg l_1 \)); we may assume \( l_2 \rightarrow \infty \). The initial temperature is constant (or equal to zero). Starting from time \( \tau = 0 \), the temperature of the outside surface changes according to an harmonic law, \( t(0, r) = t_m \sin \omega \tau \); \( t(\infty, r) = t_c \).

The solution of (7) has the form [6]:

\[
\begin{align*}
t_2(x, \tau) &= t_c = \exp \left[- \frac{\tau}{\omega^2 a_2 l_1 - \omega^2 a_2 (x - l_1)} \right] \times \frac{1 - h}{\Delta^1_{x}} \sin \left[ \omega \tau - \frac{\omega^2 a_2 (x - l_1)}{\omega^2 a_2 l_1 - \omega^2 a_2 (x - l_1)} - \beta \right] \\
\Delta_{x} &= 1 - 2h \exp \left[- 2 (l_1 - x) \sqrt{\omega^2 a_1} \cos 2 (l_1 - x) \sqrt{\omega^2 a_1} + h^2 \exp \left[- 4 (l_1 - x) \sqrt{\omega^2 a_1} \right] \right] \\
\beta &= \beta_{x=-0}.
\end{align*}
\]

For the heat fluxes we obtain

\[
\begin{align*}
q_1(x, \tau) &= \left( \frac{1 + 2h \exp \left[- 2l_1 + 2x \right]}{\sqrt{\omega^2 a_1}} \cos 2 (l_1 - x) \times \frac{\sqrt{\omega^2 a_1} + h^2 \exp \left[- 4l_1 + 4x \right]}{\omega^2 a_1} \right) \{ 1 - 2h \exp \left[- 2l_1 \times \frac{\sqrt{\omega^2 a_1} + h^2 \exp \left[- 4l_1 + 4x \right]}{\omega^2 a_1} \right] \}^{1/2} \times \exp \left(- \frac{\tau}{\omega^2 a_1} \right) \sin \left[ \omega \tau - \frac{\omega^2 a_1 (x - \beta)}{\omega^2 a_1 (l_1 - \beta)} + \beta / 2 \right] \\
q_2(x, \tau) &= \omega_2 a_1 \cos 2 l_1 \sqrt{\omega^2 a_1} + h^2 \exp \left[- 4l_1 + 4x \right] \right) \times \left[ 1 - 2h \exp \left[- 2l_1 + 2x \sqrt{\omega^2 a_1} \cos 2 l_1 \sqrt{\omega^2 a_1} + h^2 \exp \left[- 4l_1 \right] \times \frac{\sqrt{\omega^2 a_1} + h^2 \exp \left[- 4l_1 + 4x \right]}{\omega^2 a_1} \right] \}^{1/2} \sin \left[ \omega \tau - \frac{\omega^2 a_1 l_1 - \omega^2 a_1 (x - l_1) - \beta + \beta / 2} {\omega^2 a_1 (l_1 - l_1)} \right].
\end{align*}
\]

**Three-layer wall.** The temperature at the outside surface varies according to an harmonic law \( t(0, r) = t_0 + t_m \cos \omega \tau \), and heat transfer takes place at the inside surface according to Newton's law, \( \lambda_0 (\partial t_0 / \partial x) = -a(t_0 - t_1) \big|_{x=l_2} \).

The temperature of the inside air remains constant. The initial temperature is taken as the temperature established in the wall when the temperature of the outside surface \( t_0 = t_0 + t_m \), and heat transfer at the inside surface is governed by Newton's law. The solution of (7) has the form [8]:

\[
\begin{align*}
t_1(x, \tau) &= t_{0x} + t_m A_1(x, \omega \cos (\omega \tau - \gamma + \beta_{1x}); \\
t_2(x, \tau) &= t_{0x} + 2K_m A_2(x, \omega \cos (\omega \tau - \gamma + \beta_{2x}); \\
t_3(x, \tau) &= t_{0x} + 4K_1 K_m A_3(x, \omega \cos (\omega \tau - \gamma + \beta_{3x}).
\end{align*}
\]