A solution is given for the deflection of the axis of two-dimensional and circular jets by an entraining stream. Design formulas are obtained and compared with experiment.

The problem of determining the shape of a jet injected into an entraining stream is part of the general problem of the mixing of streams of gas, which owing to its great practical interest is currently receiving the attention of many researchers.

Suppose a jet of gas with the initial parameters $p_0, v_0$ is injected at an angle $\alpha_0$ to the axis through an aperture in the side wall of a channel in which another gas with density $p_w$ is flowing at the uniform velocity $w$. The injected jet will be deflected by the stream. It is required to find the curve followed by its axis.

The analytical solutions of this problem given in the literature may all be divided into two groups. The first group contains the purely kinematic solutions of [1-4], in which some form of the method of geometric superposition of the jet and the stream is used. However, although these solutions are valid for ideal fluids and potential flow, their application to real fluids can be regarded only as a coarse approximation, and therefore the results obtained show poor agreement with experiment, as pointed out in [5].

The second group of analytic solutions of the problem contains the dynamic solutions of [6-8], based on an examination of the forces exerted on the jet by the stream. In [7] the jet is regarded as an airfoil on which the aerodynamic force of the stream acts. Equating the normal component of the aerodynamic force to the centrifugal force, the author calculates the radius of curvature at each point of the trajectory and then finds the equation of the axis of the jet, which, in the particular case of a circular jet with $\alpha_0 = \pi/2$, takes the form:

$$g = 14.4 \frac{p_0 v_0^2}{c_n p_w w^2} \log \left[ 1 + 0.1 \frac{x}{d_0} \left( 1 + 20 \frac{d_0}{x} \right) \right].$$

It is known from experimental aerodynamics that the value $c_n$ is of the order of unity. The author recommends using $c_n = 3$. However, formula (1) then gives a large discrepancy with experiment (up to 300%). To make this formula agree with the experimental airfoil drag data, $c_n$ must be increased by a factor of 25-30, i.e., to values that would be hard to account for theoretically.

In this paper an attempt is made to find the shape of the jet axis in the stream from the value of the drag represented by the jet. Let us first consider an infinite two-dimensional jet (Fig. 1). Since a free jet is isobaric, the projection of its momentum on the $y$ axis remains unchanged:

$$p_0 v^2 S_0 \sin \alpha = p_0 v_0^2 S_0 \sin \alpha_0.$$  \hspace{1cm} (2)

Hence we obtain the first equation of motion of a jet issuing from an infinite slot:

$$\frac{dy}{dt} = v \sin \alpha = p_0 v_0^2 S_0 \sin \alpha_0 (p_0 v S_n)^{-1}.$$  \hspace{1cm} (3)

Let us consider a point $M$ on the jet axis with coordinates $x, y$ and calculate the pressure of the stream on a section of the jet from 0 to $M$ per unit width of the jet. Let us assume that the aerodynamic force $P$ of the flow acts only along the $x$ axis and is proportional to the velocity head:

$$P = \frac{1}{2} c_x y p_w w^2.$$  \hspace{1cm} (4)

The law of change of momentum takes the form:

$$p_0 v^2 S_0 \cos \alpha - p_0 v_0^2 S_0 \cos \alpha_0 = P,$$  \hspace{1cm} (5)

Fig. 1. Diagram of jet deflected by a flow of gas.
from which we obtain the second equation of motion of the jet:

\[
\frac{dx}{dt} = v \cos \alpha = \frac{1}{2} \left( c_x \rho_w \omega^2 + 2c_{\omega_0} S_{\rho_0} \cos \alpha_0 \right) (\rho_\omega S_\omega)^{-1}.
\] (6)

Dividing (6) by (3), we obtain the differential equation of the jet axis

\[
\frac{dx}{dy} = \frac{1}{2} c_x \rho_w \omega^2 (S_{\rho_0} \sin \alpha_0 \rho_{\omega_0} v_0^2)^{-1} + \cot \alpha,
\] (7)

where \( S_{\rho_0} = b_0 \cdot 1 \). The equation of the axis of the deflected jet is found by integrating (7) with the boundary conditions \( x \big|_{y = 0} = 0 \):

\[
\frac{x}{b_0} = \frac{1}{4} c_x \rho_w \omega^2 \left( \rho_{\omega_0} v_0^2 \sin \alpha_0 \right)^{-1} \left( \frac{y}{b_0} \right)^2 + \frac{y}{b_0} \cot \alpha_0.
\] (8)

In Fig. 2, (6), with \( c_x = 5 \) and \( \alpha_0 = \pi/2 \), is compared with the experimental data of [5] for a two-dimensional jet. An examination of Fig. 2 shows that, for \( \frac{\rho_w \omega^2}{\rho_\omega v_0^2} = 0.0025 \) and 0.08, the experimental data fall sufficiently close to the theoretical curve. Some deviation (up to 20\%) is observed for \( \frac{\rho_w \omega^2}{\rho_\omega v_0^2} = 0.01 \).

Let us now proceed to solve the problem of the deflection of the axis of a jet that is circular in section. It has been established [7] from an analysis of experimental data that, under the action of the stream, the jet section becomes distorted even at a small distance (\( l/d_0 \approx 1.5 \)) from the nozzle, acquiring a horseshoe shape with a ratio of the sides \( h/h_0 \) of the order of 1:5. The width of the horseshoe increases approximately in proportion to the distance from the nozzle:

\[
h = h_0 + c_1 l,
\] (9)

where \( h_0 \) is the nominal width of the initial section. We shall assume, as in [7], that the initial jet section is an ellipse equal in size to a circle of the initial diameter \( \delta_0 = d_0 \) and that for \( h_0 = 5 \delta_0, \delta_0 = 0.45d_0, h = 2.25d_0 \), whence \( S_{\rho_0} = \frac{1}{4} \pi d_0^2 \). Assuming that the coefficient of angular expansion of the deflected jet of horseshoe section is the same as that for a straight jet \( c_1 = 0.22 \), we obtain

\[
h = 2.25d_0 + 0.22 l.
\] (10)

Consider a point with coordinates \( x, y \) on the axis of the jet. From the condition that the projection of the jet momentum

![Fig. 2. Comparison with the experimental data of [5] for a two-dimensional jet: (a) 4 — according to (8) and for a circular jet, (b) 4 — according to (20), 5 — according to (1) with \( c_n = 3 \), I, II, and III correspond to velocity head ratios of 0.0025, 0.01 and 0.08): \( 1 - \rho_w \omega^2/\rho_\omega v_0^2 = 0.0025; 2 - 0.01; 3 - 0.08 \)]