ANALYSIS OF THE FEASIBILITY OF DESCRIBING UNSTEADY HEAT CONDUCTION PROCESSES IN DENSE DISPERSE SYSTEMS BY DIFFERENTIAL EQUATIONS ACCOUNTING FOR PHASE INTERACTION CHARACTERISTICS

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The solution of a set of differential equations for unsteady heat conduction of a disperse medium is analyzed and a comparison is presented with experimental data from several authors.

The set of differential equations for unsteady heat conduction in a disperse medium [1] is based on a model representation which calls for imaging of the most essential portion of the process which is extremely difficult to describe in a rigorous mathematical manner. In selection of the model it is not possible to obtain an a priori quantitative estimate and, at times, even a qualitative estimate of the validity of simplifying assumptions. Intuitive physical considerations which form the basis for model selection, and are written in the form of differential equations, must be verified by analysis of the solution and comparison with experimental data. Often, however, even comparison of experimental and theoretical data is limited. Here it is necessary to note that agreement between theoretical and experimental values in a given case may be fortuitous and hence, the model may only be considered for interpolation or computation. A model based on qualitative approximate estimates of the complex real processes may be considered as having physical basis if the relationships obtained not only satisfy the experimental data over a wide range of variables but also do not lose their physical significance when extrapolated beyond the limits encompassed by experiment.

1. Determination of Computational Relationships. Accounting for the fact that, for the disperse systems under consideration (disperse solid phase - gas), the thermal conductivity of the solid phase is 4 to 5 orders of magnitude lower than that of the system as a whole, we simplify the initial set of differential equations for unsteady heat conduction in a disperse medium and represent them in the form

\[ c_1 \rho_1 \frac{\partial \theta_1}{\partial t} = \alpha S (\theta_2 - \theta_1), \]  

\[ c_2 \rho_2 \frac{\partial \theta_2}{\partial t} = \lambda_2 \frac{\partial^2 \theta_2}{\partial x^2} - \alpha S (\theta_2 - \theta_1). \]  

This simplified set of equations corresponds to a model of unsteady heat conduction in a disperse medium in which, in contrast to the initial one, [1, 8], one of the phases (namely the solid phase) is considered only as a thermal sink while the second (gas) phase determines the molecular transport of internal energy within the system.

From sufficiently general physical considerations it follows that, for disperse systems of the type, disperse solid phase - gas, the simplified model apparently corresponds closely to the physical nature of the process. We note also that these considerations are confirmed by some of the results of this work.

To find the solution to the set (1, 2) for the boundary conditions

\[ \theta_1(x, 0) = \theta_2(x, 0) = 0; \theta_2(0, t) = \theta_0; \theta_1(\infty, t) = \theta_2(\infty, t) = 0 \]  


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we transform it to nondimensional form. Since the surface area of the particles per unit volume may be determined as \( S = 6(1 - e)/d_p \) \((m^2/m^3)\), we write

\[
\xi = \sqrt{\frac{\alpha S}{\lambda_2}} = \sqrt{6(1 - e)Nu^* \frac{x}{d_p}} = \frac{x}{d_p},
\]

(4)

\[
t = \frac{\alpha S}{c_0 \rho_1} = 6(1 - e)Nu^*Fo = A^2Fo,
\]

(5)

\[
\theta_1(\xi, t) = \frac{\theta_1(x, t)}{\theta_0} \text{ and } \theta_2(\xi, t) = \frac{\theta_2(x, t)}{\theta_0}.
\]

(6)

It follows from (4) and (5) that \( \xi \) is a measure of the spacing in kernel diameters \( (\xi \sim x/d_p) \) and \( t \) is proportional to the Fourier number.

In new variables the initial equations (1) and (2) take the form

\[
\frac{\partial \theta_1}{\partial t} = \theta_2 - \theta_1,
\]

(7)

\[
\frac{\partial \theta_2}{\partial t} = \frac{\partial^2 \theta_2}{\partial \xi^2} - \theta_3 + \theta_1,
\]

(8)

where \( \mu = \frac{c_2 \rho_2}{c_1 \rho_1} \) is a nondimensional parameter \((\mu \ll 1)\). The initial and boundary conditions are written in the following form:

\[
\theta_1(\xi, 0) = \theta_2(\xi, 0) = 0; \quad \theta_2(0, t) = 1 \quad \text{and} \quad \theta_1(\infty, t) = \theta_3(\infty, t) = 0.
\]

(9)

The thermal flux at the boundary of a surface with constant temperature and disperse medium is

\[
q(t) = -\lambda_2 \frac{\partial \theta_2}{\partial \xi} \Big|_{\xi=0} = -V \lambda_2 \alpha^* \frac{\partial \theta_2}{\partial \xi} \Big|_{t=0}
\]

(10)

and the heat transfer coefficient is correspondingly

\[
a(t) = \frac{V \lambda_2 \alpha^*}{\lambda_2} \frac{\partial \theta_2}{\partial \xi} \Big|_{t=0}.
\]

(11)

For convenience in comparison with experimental data, we introduce a nondimensional, time-dependent heat transfer coefficient

\[
N(t) = \frac{a(t)}{Nu} = \frac{Nu}{6(1 - e)Nu^*Nu} = \frac{Nu}{A}
\]

(12)

and, consequently,

\[
N(t) = -\frac{\partial \theta_2(\xi, t)}{\partial \xi} \Big|_{t=0}.
\]

(13)

If we solve the set of differential equations (7) and (8) under conditions (9) and place them in (13), we obtain the desired dependence of Nusselt number on Fourier number. However, using operational techniques, relationship (13) may be found without prior solution of equations (7) and (8).

We introduce the Laplace transforms of the functions \( \theta_1 \) and \( \theta_2 \). Then Eqs. (7) and (8) take the form

\[
sY_1 = Y_2 - Y_1 \quad \text{and} \quad \mu sY_2 = \frac{dY_2}{d\xi} - Y_2 + Y_1
\]

(14)

with boundary conditions

\[
Y_2(0, s) = \frac{1}{s} \quad \text{and} \quad Y_2(\infty, s) = 0.
\]

(15)

Eliminating \( Y_1 \) from Eq. (14), we obtain

\[
\frac{dY_2}{d\xi} = \left(1 + \mu s - \frac{1}{1 + s}\right) Y_2.
\]

(16)

Solution of Eq. (16) under conditions (15) has the form